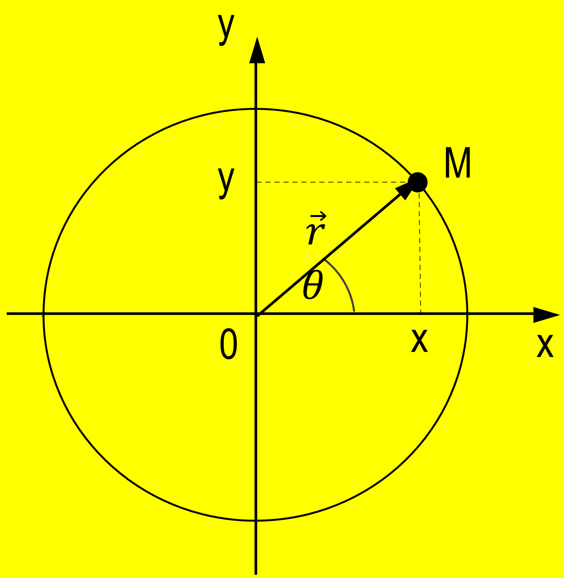
CIRCULAR

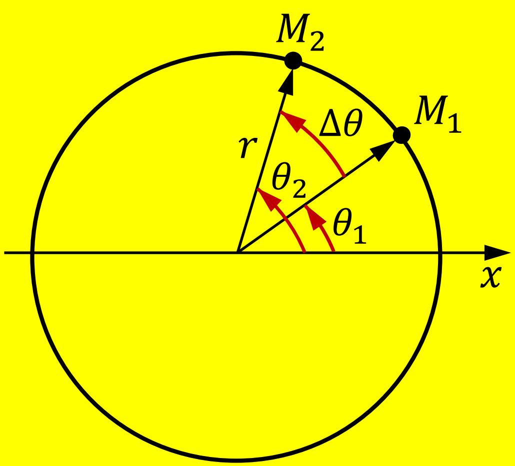
MOTION



CIRCULAR MOTION

Circular motion is the motion of an object around a path (circle). It can be uniform or non-uniform circular motion. Circular motion can be describe as one-dimensional motion. The material point is at a constant distance r from the origin 0.

The position of the material point M along the circular path is uniquely derminated by the angle θ,this is angle between the radius-vector and the x axis. In this example , the value of angle θ is changing, while the value of the radius remains the same. Angular quantities that are used in circular motion of material points are: described angle, angular displacement, angular velocity and angular acceleration.



ANGULAR DISPLACEMENT, DESCRIBED ANGLE

The material point travel from M_1 to M_2 as it does in the picture, radius-vector describes an angle $\Delta\theta$.

M_1 and M_2 are the initial and final position of radius-vector of material point

$\Delta\theta = \theta_2$-θ_1 is angular displacement of material point

To know how far an material point has moved round the circle, we need to know the angle θ.

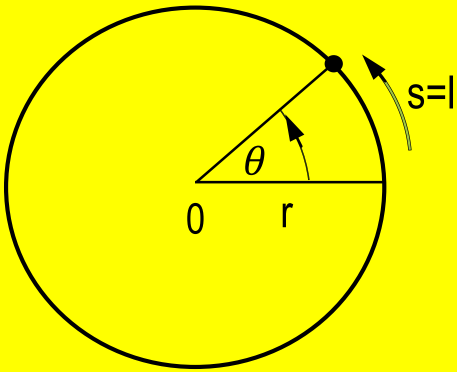
Angular displacement of material point is angular distance between the radius-vector material points in the initial and final position.

Described angle is the total angle which describe the material point in circular motion.

if the material point describes less than one full revolution angular displacement and described angle are equal

if the material point describes more than one full revolution described angle is greater than the angular displacement

When dealing with circles and circular motion. It is more convenient to measure angles and angular displacement in units called radians rather than in degrees. If an object moves a distance s around a circular path of radius r (picture), its angle θ in radians is defined as fallows:



$$angle(in\,radians) = \frac{length\ of\ arc}{radius} \Rightarrow \theta = \frac{l}{r}$$

The size of an angle depends on the radius and length of the arc.

Since s and r are distance measured in metres, it fallows that the angle θ is simply a ratio. It ia a dimensionless quantity.

If an object moves all the way round the circumference of the circle, it moves distance of $2\pi r$. We can calculate its angular displacement in radians:

$$\theta = \frac{circumference}{radius} = \frac{2\pi r}{r} = 2\pi rad$$

An angle of 360° is equivalent to an angle of 2π radians.If we want to transform the angle from degrees to radians or viceversa we use proportion: $360^{\circ}: 2\pi rad = \theta(^{\circ}): \theta(rad)$

Example 1:

If $\theta = 60^{\circ}$, what is the value of θ in radians?

Solution: $360^{\circ} : 2\pi rad = 60^{\circ} : \theta$

$$\theta = 60x\frac{2\pi rad}{360} = \frac{\pi}{3} rad = 1{,}05\, rad$$

Example 2:

If $\theta = 1 rad$, what is the value of θ in degrees?

Solution: $\theta = \ 57{,}3^{\circ}$

Based on the definition angle, it is easy to establish a connection between the distance and the angle described in a circular motion. Distance is equal lengh of arc.

$$\theta = \frac{s}{r}$$

$$s = r\theta$$

ANGULAR VELOCITY

An object's **average angular velocity** during a time interval Δt is its angular displacement $\Delta\theta$ divided by Δt:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$ **SI unit: radian per second (rad/s)**

Estimate

$\omega = \Delta\theta/\Delta t$

for these windmills if shutter speed = 1/60 sec.

Remember ω is in radians/second!

If towers are 20 m. high, estimate speed of blade tips.

By analogy with linear velocity, instantaneous angular velocity ω, is defined as the limit of the average velocity at the time interval approaches zero:

$$\omega = \frac{\Delta\theta}{\Delta t}, \Delta t \rightarrow 0$$ **SI unit: radian per second (rad/s)**

We take ω to be positive when ω is increasing (counterclockwise motion), and negative when ω is decreasing (clockwise motion). When the angular velocity is constant, the instantaneous angular velocity is equal to the average angular velocity.

► linear velocity and angular velocity

If the material point moves along a circular line of radius r, the material point of infinite little time Δt describes the angle $\Delta\theta$ and distance equal to the length of arc Δl.

$$v = \frac{\Delta l}{\Delta t}, \omega = \frac{\Delta\theta}{\Delta t}$$

$$\Delta l = r\Delta\theta$$

This show that the speed v of an object trevelling around a circle depends on two quantities:
Its angular velocity ω and its distance from the centre of the circle r.

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega$$

UNIFORM CIRCULAR MOTION

Uniform circular motion can be described as the motion of an object in a circle with a constant linear velocity ($v = const$).

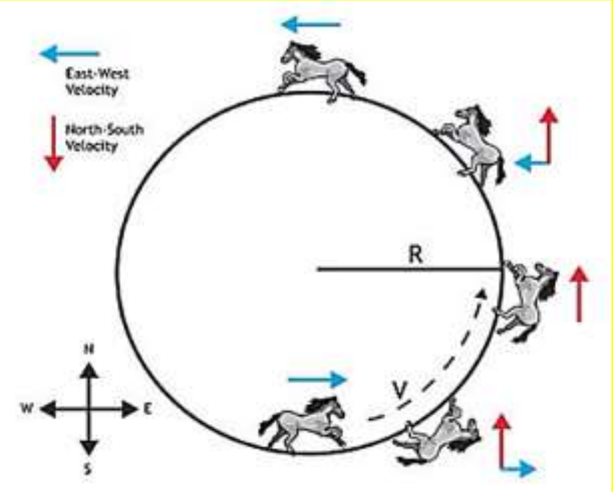
Uniform circular motion can be described as the motion of an object in a circle with a constant angular velocity ($\omega = const$).

The distance which cross the material point for some time is equal to the product of relating velocity and that time.

$$s = v \cdot t$$

The angle that describes the material point for some time is equal to the product of the angular velocity and that time

$$\theta = \omega \cdot t$$



The velocity vector shows the direction of motion at any point on the circle.

If T is the time in which object makes one full revolution. We call T the period of the motion. Since the speed is constant and the object covers a distance of $2\pi r$ in a time t, it follow that $v = \frac{2\pi r}{T}$

We may also note that the object sweeps out an angle of 2π radians in a time equal to the period, so we define the angular speed of the object by

$$angular\ velocity = \frac{angle\ swept}{time\ taken} \Rightarrow \omega = \frac{2\pi}{T}$$

Frequency is number of revolutions per unit of time. $f = \frac{n}{t} \to \left(\frac{n=1}{t=T}\right) \to f = \frac{1}{T}$.

So frequency is the reciprocial of the period. **SI unit: hertz (Hz)**

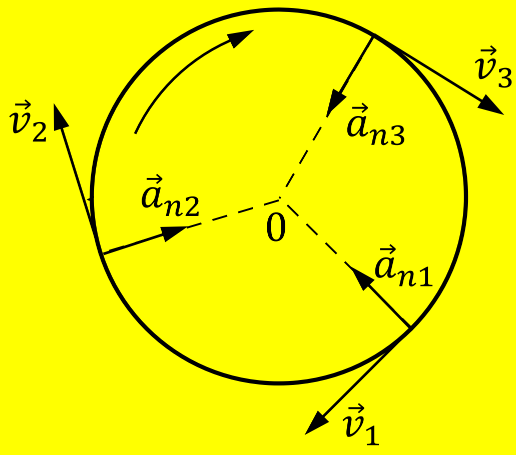
NORMAL ACCELERATION (CENTRIPETAL ACCELERATION)

Object moves along a circle of radius r with constant magnitude of velocity v, (angular and tangential acceleration are equal to zero) experience normal acceleration that has magnitude given by $a_n = \frac{v^2}{r}$ and is directed toward the centre of the circle.

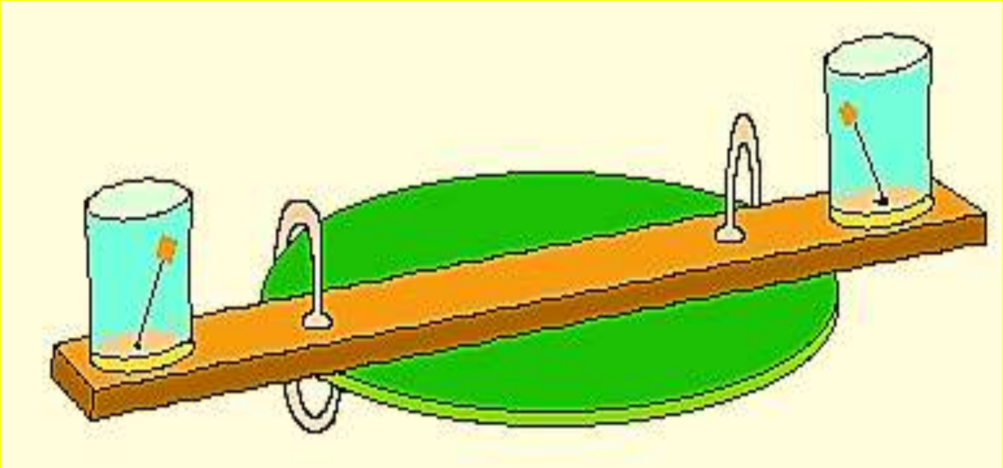
SI unit: meter per second squared (m/s^2)

$a_n = \frac{v^2}{r}$	$v = r \cdot \omega$	$a_n = \frac{r^2\omega^2}{r} = r\omega^2$

Normal acceleration characterizes the change in the direction of the velocity $\vec{v}$.



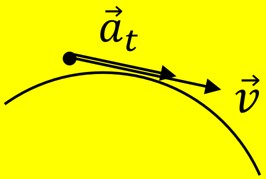
The normal acceleration vector is normal to the velocity vector.



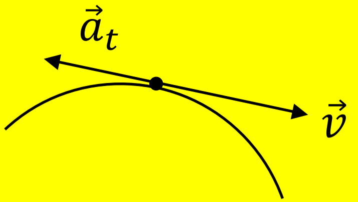
The cook of a water accelerometer point inward towards the center of the circle when placed upon a rotating platform, thus indicating an inward acceleration for circular motion.

TANGENTIAL ACCELERATION

If the magnitude of the velocity vector changes,we have tangential acceleration.This is vector directed along the vector if the velocity is increasing (picture a), and opposite if the velocity is decreasing (picture b). The magnitude of the tangential acceleration is given by: $a_t = \frac{\Delta v}{\Delta t}$



picture (a)



picture (b)

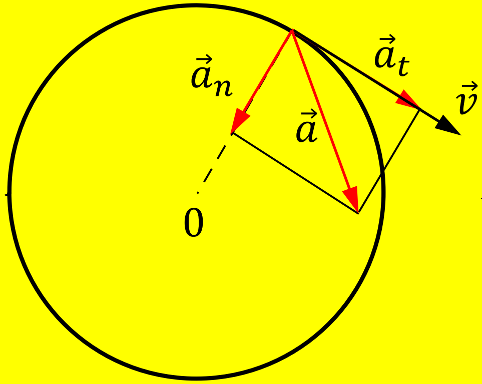
The golfer example problem

When he is getting ready to swing, the tangential acceleration is zero and as he swings the driver down towards the ball, the tangential acceleration increases Hence there is an tangential acceleration (Same holds true for angular acceleration).

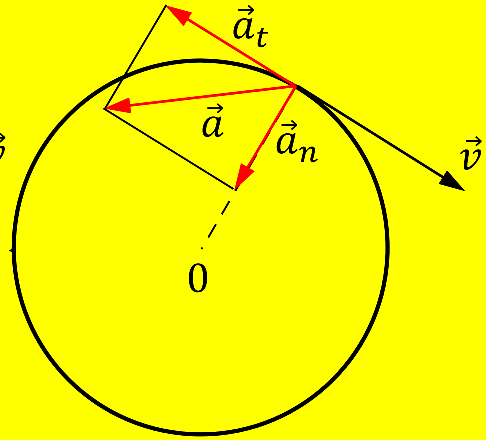
TOTAL ACCELERATION

The foregoing derivations concern circular motion at constant speed. When an object moves in a circle but is speeding up (picture b) or slowing down (picture a), a tangential component of acceleration, $a_t = r\alpha$, is also present. Because the tangential and centripetal components of acceleration are perpendicular (normal) to each other, we can find the magnitude of the **total acceleration** with the Pythagorean theorem:

$$a = \sqrt{a_t^2 + a_n^2}$$



picture a



picture b

ANGULAR ACCELERATION

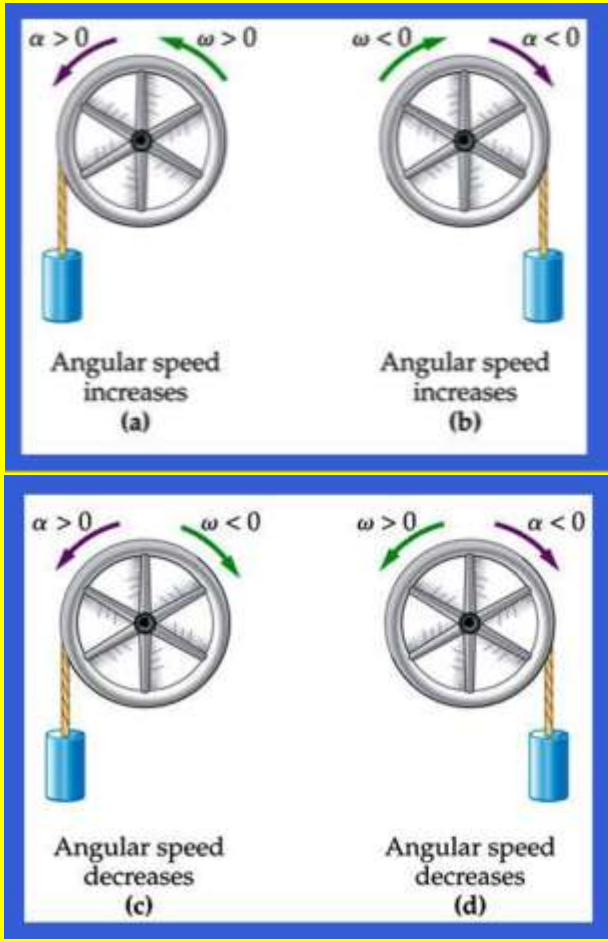
An object's average angular acceleration α_{av} during the time interval Δt is the change in its angular velocity $\Delta\omega$ divided by Δt:

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$ **SI unit: radian per second squared (rad/s^2)**

The instantaneous angular acceleration α is the limit of the average angular acceleration $\Delta\omega/\Delta t$ as the time interval Δt approaches zero:

$$\alpha = \frac{\Delta\omega}{\Delta t}, \Delta t \rightarrow 0$$ **SI unit: radian per second squared (rad/s^2)**

If the angular velocity increases, the angular acceleration is positive
If the angular velocity decreases, angular acceleration is negative



► tangential and angular acceleration

The tangential acceleration of a point on a rotating object equals the distance of that point from the axis of rotation multiplied by the angular acceleration.

$$a_t = \frac{\Delta v}{\Delta t}, \alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta v = r\Delta\omega$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{r\Delta\omega}{\Delta t} = r\alpha$$

Linear motion		Curcular motion	
distance (s)		angle (θ)	
displacement (Δx)		angular displacement ($\Delta\theta$)	
velocity (v)	$v = \frac{\Delta x}{\Delta t}$	angular velocity (ω)	$\omega = \frac{\Delta\theta}{\Delta t}$
acceleration (a)	$a = \frac{\Delta v}{\Delta t}$	angular acceleration (α)	$\alpha = \frac{\Delta\omega}{\Delta t}$

NON-UNIFORM CIRCULAR MOTION-FORMULAS

Uniformly-accelerated motion		Uniformly-accelerated circular motion	
velocity as a function of time	$v = v_0 + at$	angular velocity as a function of time	$\omega = \omega_0 + \alpha t$
distance as a function of time	$s = v_0 t + \frac{at^2}{2}$	angle as a function of time	$\theta = \omega_0 t + \frac{\alpha t^2}{2}$
velocity as a function of distance	$v = \sqrt{v_0^2 + 2as}$	angular velocity as a function of angle	$\omega = \sqrt{\omega_0^2 + 2\alpha\theta}$

Uniformly-deccelerated motion		Uniformly-deccelerated circular motion	
velocity as a function of time	$v = v_0 - at$	angular velocity as a function of time	$\omega = \omega_0 - \alpha t$
distance as a function of time	$s = v_0 t - \frac{at^2}{2}$	angle as a function of time	$\theta = \omega_0 t - \frac{\alpha t^2}{2}$
velocity as a function of distance	$v = \sqrt{v_0^2 - 2as}$	angular velocity as a function of angle	$\omega = \sqrt{\omega_0^2 - 2\alpha\theta}$

PROBLEMS

I. Convert the following angles from degrees into radians: $30^{\circ}, 60^{\circ}, 90^{\circ}$.

II. Convert the following angles from radians into degrees: 45rad, πrad, $\frac{\pi}{2}$rad.

III. Find the angular velocity of:

a) daily rotation of the earth

b) a minute hand watch

c) a second hand watch

d) if the length of the second hand is 1,8cm, calculate the speed of the tips of the second hand as it moves round

IV. A point moves along a circle of radius 5cm. It makes 30 revolutions in 10s. Find linear velocity and normal acceleration of a point.

V. A point moves along a circle of radius 20cm with period 2s. Find linear velocity and angular velocity of a point.

VI. A car travels around a 90° bend in 15s. The radius of the band is 50m.

a) Determinate the angular velocity of the car

b) Determinate the linear velocity of the car

VII. Find the linear velocity and centripetal acceleration of an artificial satellite of the Earth rotating along a circular orbit with the period of revolution T=98 min .Its orbit is at a distance of 200 km from"the Earth's surface. The radius of Earth is $6{,}4 \cdot 10^{6}$m.

VIII. Tangential acceleration and normal acceleration of a point are: 0,3 $\frac{m}{s^2}$ and 0,4$\frac{m}{s^2}$. Find total acceleration of a point.

IX. A point moves along a circle. Normal acceleration of a point is 2$\frac{m}{s^2}$. If the vector of the total acceleration of a point on the rim forms an angle of 60° with the direction of the linear velocity of this point, find total acceleration of a point.

X. The drum of a washing machine spins at a rate 1200rpm (revolution per minute)

a) Determinate the number of revolution per second of the drum

b) Determinate the angular velocity of the drum

XI. In one minute after it begins to rotate a flywheel acquires a angular velocity corresponding 720rpm. Find the angular acceleration of the wheel.

XII. When braked a uniformly retarded wheel reduce its angular velocity from 10$\pi\frac{rad}{s}$ to 6$\pi\frac{rad}{s}$ during one minute. Find the angular acceleration of the wheels and number of revolution it computes in this time.

XIII. A shaft rotates at a constant angular velocity corresponding to the frequency 180rpm. At a certain moment the shaft is braked and begin to slow down uniformly with an angular acceleration 3$\frac{rad}{s^2}$

a) How much time will the shaft need to stop?

b) What number of revolution will it perform before stopping?

XIV. The graph the variation of the angular velocity of a point with time.

a) Draw the graph showing variation of angular acceleration with time

b) Find the angle which describe material point during all time of motion