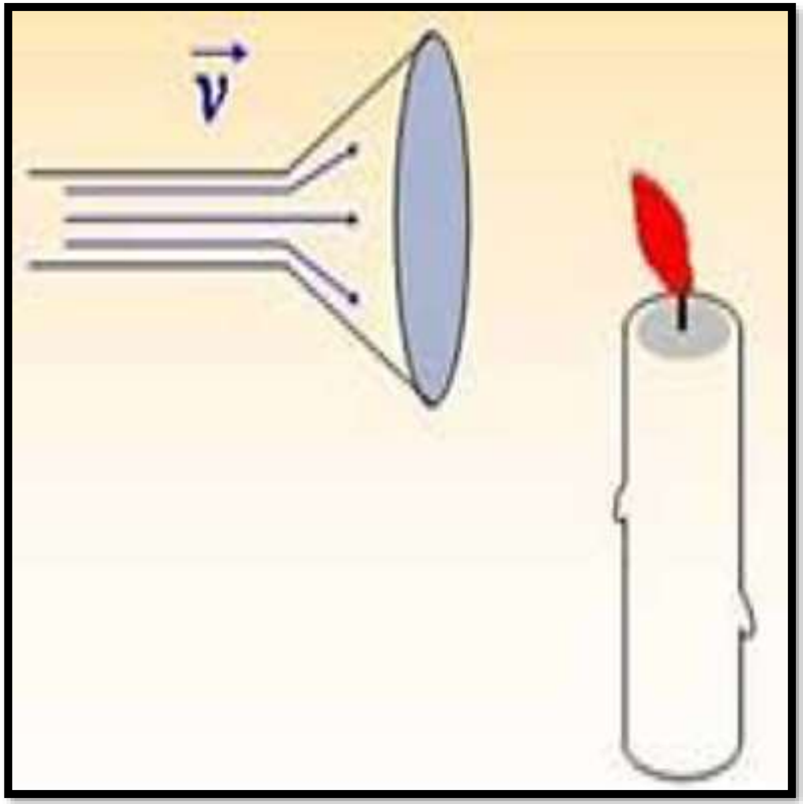
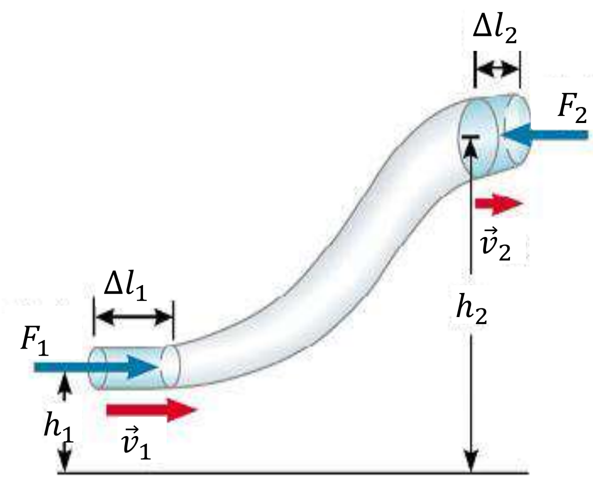
BERNOULLI'S EQUATION AND IT'S APPLICATION



BERNOULLI'S EQUATION

As a fluid moves through a pipe of varying cross section and elevation, the pressure changes along the pipe. In 1738 the Swiss physicist Daniel Bernoulli (1700–1782) derived an expression that relates the pressure of a fluid to its speed and elevation. Bernoulli's equation is a **consequence of energy conservation as applied to an ideal fluid.** In deriving Bernoulli's equation, we again assume the fluid is incompressible (density is constant), nonviscous (no internal friction), and flows in a nonturbulent, steady-state manner (velocity, density, and pressure at each point in the fluid don't change with time).



(picture 1)

A fluid flowing through a constricted pipe with streamline flow. The fluid in the section with a length of Δl_1 moves to the section with a length of Δl_2. The volumes of fluid in the two sections are equal.

Consider the flow through a nonuniform pipe in the time Δt, as in (picture 1). The force on the lower end of the fluid is $p_1 A_1$, where p_1 is the pressure at the lower end. The work done on the lower end of the fluid by the fluid behind it is:

$$W_1 = F_1 \Delta l_1 = p_1 A_1 \Delta l_1 \Rightarrow \Delta V = A_1 \Delta l_1 \Rightarrow W_1 = p_1 \Delta V$$

(where ΔV is the volume of the lower blue region in the picture)

In a similar manner, the work done on the fluid on the upper portion in the time Δt is:

$$W_2 = -F_2 \Delta l_2 = -p_2 A_2 \Delta l_2 \Rightarrow \Delta V = A_2 \Delta l_2 \Rightarrow W_2 = -p_2 \Delta V$$

The volume is the same because, by the equation of continuity, the volume of fluid that passes through S_1 in the time Δt equals the volume that passes through S_2 in the same interval. The work A_2 is negative because the force on the fluid at the top is opposite its displacement. The resultant work done by these forces in the time Δt is:

$$W = p_1 \Delta V - p_2 \Delta V \ldots . (1)$$

Part of this work goes into changing the fluid's kinetic energy, and part goes into changing the gravitational potential energy. If Δm is the mass of the fluid passing through the pipe in the time interval Δt, then the change in kinetic energy of the volume of fluid is: $\Delta E_K = \frac{1}{2}\Delta m(v_2^2 - v_1^2)$.....(2)

The change in the gravitational potential energy is: $\Delta E_P = \Delta mg(h_2 - h_1)$......(3)

Because the resultant work done by the fluid on the segment of fluid shown in picture1 changes the kinetic energy and the potential energy of the nonisolated system, we have: $W = \Delta E_K + \Delta E_P$

Substituting expressions for each of the terms gives: $p_1\Delta V - p_2\Delta V = \frac{1}{2}\Delta m(v_2^2 - v_1^2) + \Delta mg(h_2 - h_1)$

If we divide each term by ΔV and recall that $\rho = \Delta m/\Delta V$, this expression becomes

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

Rearrange the terms as follows: $\boldsymbol{p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$

This is **Bernoulli's equation**, often expressed as:

$$\mathbf{p} + \boldsymbol{\rho g h + \frac{1}{2}\rho v^2 = const}$$

Bernoulli's equation states that the sum of the external pressure *p*, hydrodynamic pressure (the kinetic energy per unit volume) $\frac{1}{2}\rho v^2$, and hydrostatic pressure (the potential energy per unit volume) $\rho g h$ has the same value at all points along a streamline.

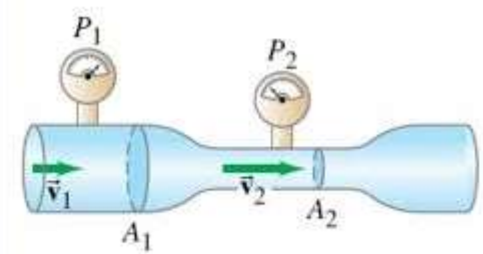
Daniel Bernoulli

An important consequence of Bernoulli's equation can be demonstrated by considering picture 2, which shows water flowing through a horizontal constricted pipe from a region of large cross-sectional area into a region of smaller cross- sectional area. Because the pipe is horizontal $h_1 = h_2$ then:

$$\boldsymbol{p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

applied to points 1 and 2 gives

$$\boldsymbol{p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2} \quad \dots (2)$$



(picture 2)

Because the water is not backing up in the pipe, its speed v_2 in the constricted region must be greater than its speed v_1 in the region of greater diameter. From Equation (2), we see that p_2 must be less than p_1 because $v_2 > v_1$ This result is often expressed by the statement that **swiftly moving fluids exert less pressure than do slowly moving fluids**..

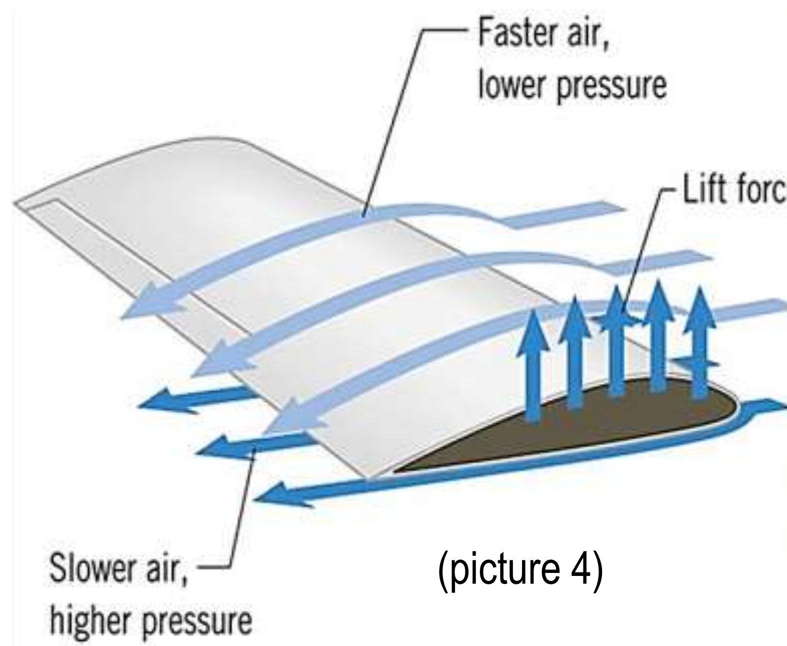
► APPLICATION OF BERNOULLI'S EQUATION

Bernoulli's equation can be seen in various places around us. Here are some example of Bernoulli's equation:

(picture 3)

A person who stands near a railway (picture 3) feels like falling into it when suddenly a train moves with a high speed passes him. It is because the velocity of air in front of him increases. According to Bernoulli's Principle, the pressure of the moving air decreases as the speed of the air increases. The higher atmospheric pressure behind pushes him forward.

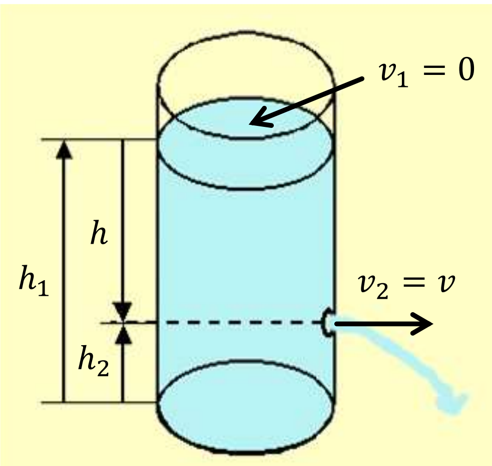
The shape of a wing (picture 4) forces air to travel faster over the curved upper surface than it does over the flatter lower surface. According to Bernoulli's equation, the pressure above the wing is lower (faster moving air), while the pressure below the wing is higher (slower moving air).
The wing is lifted upward due to the higher pressure on the bottom of the wing.



(picture 4)

TORRICELLI'S THEOREM (FLOW OF A LIQUID FROM A HOLE)

Let us apply Bernoulli's equation to the flow of a liquid from a small hole in a wide open vessel. Let us separate in the liquid a flow tube having the open surface of the liquid in the vessel as one of its cross sections and the hole through which the liquid flows out as the other one (picture 5). For each of these sections, the velocity and the height above an initial datum level may be considered the same.



(picture 5)

Consequently, we can apply. $p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$ ... (1) obtained on this assumption, to these sections.

Further, the pressure in both sections is atmospheric and therefore the same.

$$p_1 = p_2 = p_0$$

In addition, the velocity of the open surface in the wide vessel can be assumed to equal zero. $v_1 = 0$

With a view to everything said above, equation (1) can be written in the following form for this case

$p_0 + \rho g h_1 = p_0 + \frac{1}{2}\rho v^2 + \rho g h_2$ (where v is the velocity of the liquid flowing from the hole)

Cancelling ρ and introducing $h = h_1 - h_2$ (the height of the open surface of the liquid above the hole,) we get:

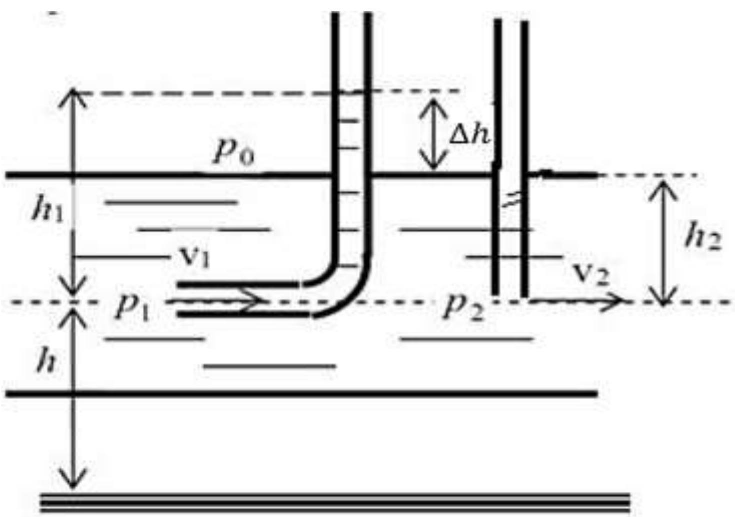
$$\boldsymbol{v = \sqrt{2gh}}$$

This formula is known as the Torricelli **formula** (after the Italian physicist Evangelista Torricelli, 1608-1647).

Thus, the velocity with which a liquid is discharged from a hole at a depth of *h* under an open surface coincides with the velocity which a body acquires in falling from the height *h.*

PITOT PIPE

Flow velocity and pressure in horizontal pipe can be measured using vertical pipes. The pitot pipe was invented by the French engineer Henri Pitot in the early 18th century. Two hollow vertical pipes(picture 6) are placed in liquid .The first tube(pitot tube) is bent at right angle into the fluid , while the second tube is placed vertically into the fluid. The moving fluid is brought to rest (stagnates) in pitot tube as there is no outlet to allow flow to continue (velocity becomes zero).



(picture 6)

However Bernoulli equation for horizontal pipe states:

$$p_1 + \rho g h + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h + \frac{1}{2}\rho v_2^2$$

$$v_1 = 0, v_2 = v$$

Which can also be written: $p_1 = p_2 + \frac{\rho v^2}{2}$

$$p_1 = p_0 + \rho g h_1 \quad p_2 = p_0 + \rho g h_2$$

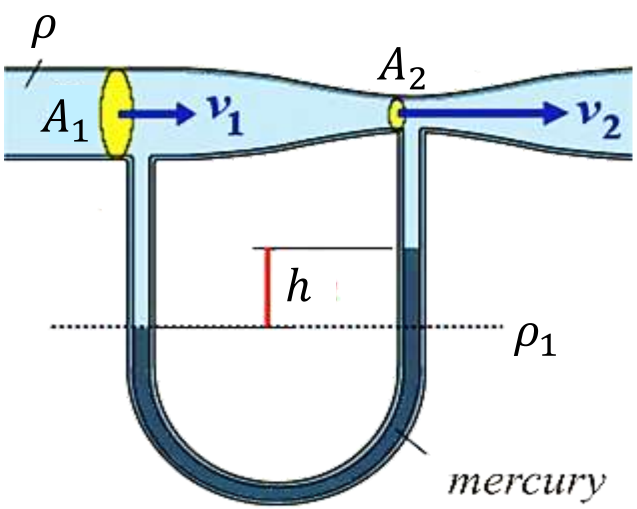
Since the difference in pressure $p_1 - p_2 = \rho g \Delta h$

Solving that for flow velocity:

$$\rho g \Delta h = \frac{\rho v^2}{2} \Rightarrow \boldsymbol{v = \sqrt{2 g \Delta h}}$$

FLUID FLOW VENTURI-METER

The Venturi-meter is a device to measure the flow speed of incompressible fluid. The basic principle on which it works is that by reducing the cross-sectional area of the flow passage. The pressure difference is measured by using a differential U-tube manometer. This pressure difference helps in the determination of rate of flow of fluid. As the inlet area (A_1) of the venturi is large than at the throat (A_2), the velocity at the throat (v_2) increases resulting in decrease of pressure (p_2). By this, a pressure difference is created between the inlet (p_1) and the throat of the venturi (p_2). The manometer contains a liquid of density ρ_1.



(picture 7)

The speed v_1 of the liquid flowing through the tube at the inlet area A_1 is to be measured from equation of continuity.: $v_1 A_1 = v_2 A_2$

speed at the throat becomes v_2 ➡ $v_2 = \frac{v_1 A_1}{A_2} \ldots (1)$

Since the difference in pressure: $p_1 - p_2 = \rho_1 g h \ldots (2)$

Then using Bernoulli's equation for horizontal pipe we get:

$$p_1 + \frac{\rho v_1^2}{2} = p_2 + \frac{\rho v_2^2}{2} \Rightarrow p_1 - p_2 = \frac{1}{2}\rho[v_2^2 - v_1^2]$$

Substituting expressions for each of the terms gives: ➡ $\rho_1 g h = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$ ➡ Solving that for speed v_1 we get:

$$v_1 = A_2 \sqrt{\frac{\rho_1}{\rho} \frac{2gh}{A_1^2 - A_2^2}}$$

It follows that the volume flow rate is equal:

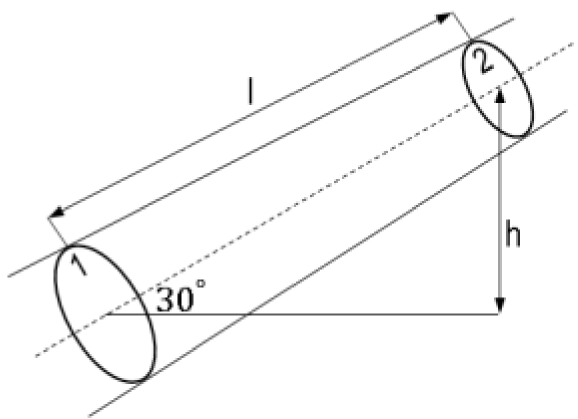
$$q_V = A_1 v_1 = A_1 A_2 \sqrt{\frac{\rho_1}{\rho} \frac{2gh}{A_1^2 - A_2^2}} = C\sqrt{h}$$

➡ The coefficient C is determined experimentally. It is called the venturi coefficient.

PROBLEMS

1.Through the horizontal pipe with the varying cross section, flows the water . The volume flow rate of water is 10 l/s. Find the difference in the pressure in the wider and narrow part of the pipe if the radius of the wider part is 4 cm, and the narrow part is 1 cm.

2.Through the pipe shown in the picture 1, flows the liquid with density 800$\frac{kg}{m^3}$. Find the difference in the pressure between sections 1 and 2 is if the distance between their centers is l=50 cm and the velocity at the section 1 is 2 m / s. Cross-sectional areas are:A_1=5cm^2 and A_2=2cm^2 .

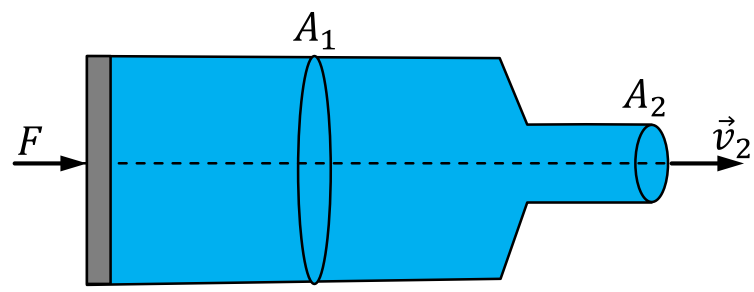


picture 1

3.An open tank is filled with water to a height h=1,5m.A small hole (at a height of h_1=0,5m from the bottom of the tank) is opened up and water begins to flow freely out. Find the horizontal distance traveled by the water before hitting the ground?

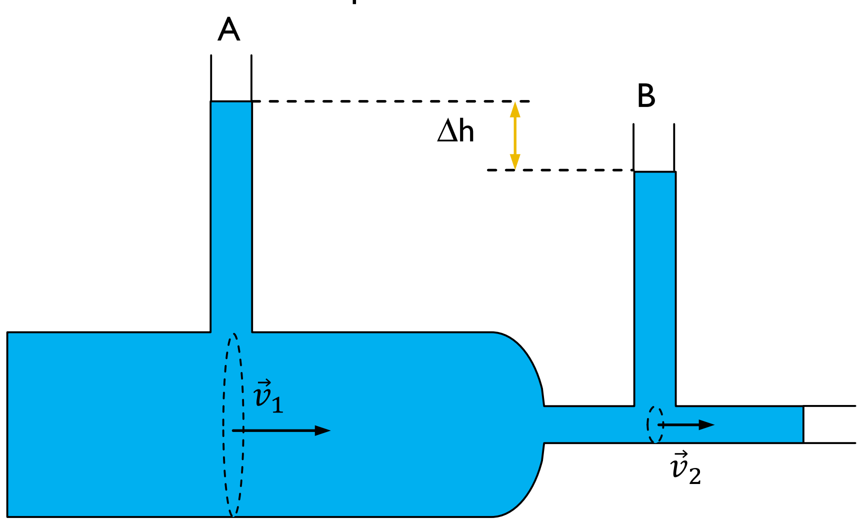
4. The open tank has two holes makes in the wall, first at a height $h_1 = 10\text{cm}$ (measured from the bottom) and the second at a height h_2 =30cm measured from the bottom). At what height h must be the water level in the tank, if we want the water jets to steam out of both holes to obtain the same range?

5.The radius of the wider part of the medical syringe is $r_1 = 1\text{cm}$, and the radius of the narrow part is r_2=1mm. Find the velocity of the water as it leaves the syringe if 10N force act on the piston. (picture 2)



picture 2

6. Water flows through the horizontal pipe with varying cross section shown in picture 3. Radius of wider part of pipe is 10cm and radius of narrow part of pipe is 5cm. Find the velocity in narrow part of pipe if $\Delta h = 25cm$ is difference in water level in two vertical pipes A and B.



picture 3