

CURVILINEAR



MOTION

## CURVILINEAR MOTION

The motion of an object moving in a curved line is called *curvilinear motion*. Many things move in circle. Curvilinear motion is different from the linear motion.

- 1) the wheels of a car or a bicycle
- 2) the hands of a clock
- 3) the spinning DVD in a laptop (picture 1)
- 4) the drum of a washing machine



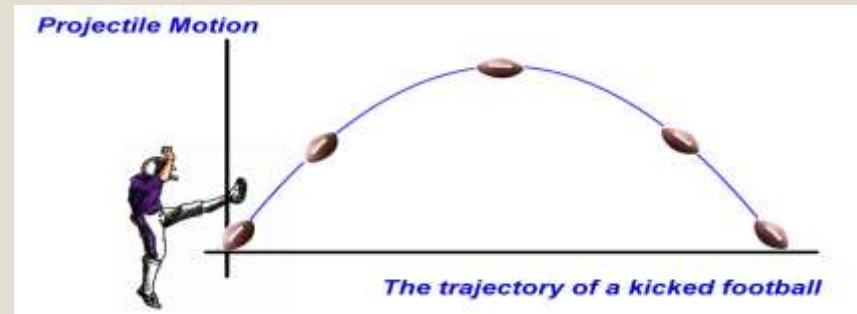
picture 1

Some other examples of curvilinear motion include...

- The trajectory of a motor cyclist in the air (picture 2)
- The trajectory of a kicked football (picture 3)



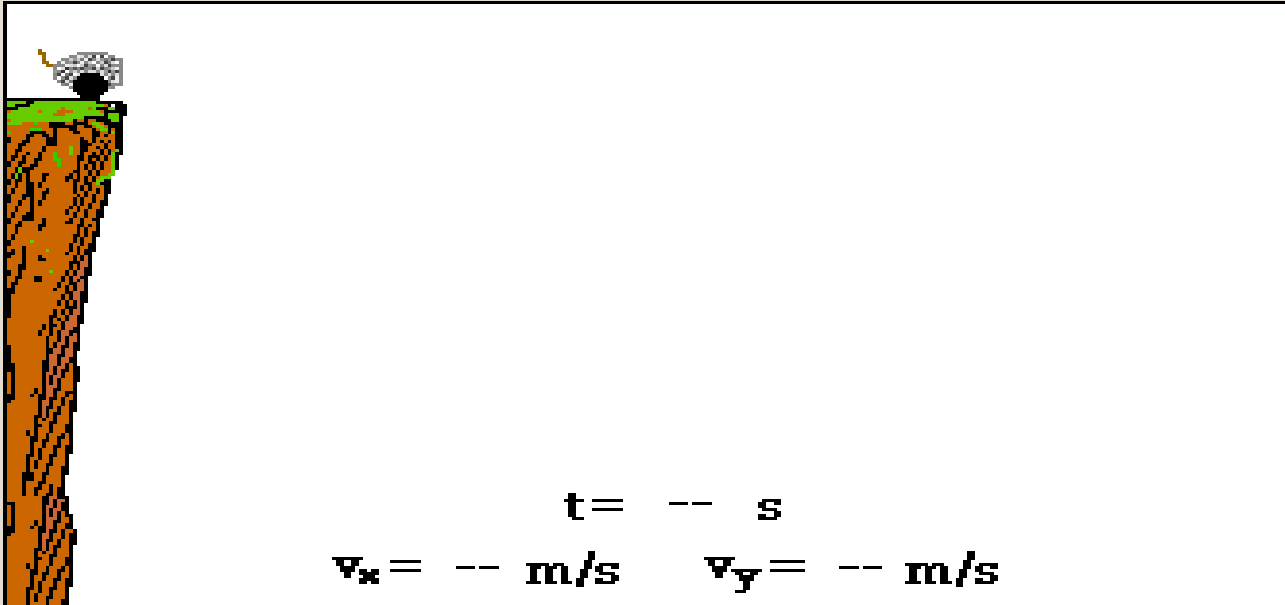
picture 2



picture 3

## HORIZONTALLY LAUNCHED PROJECTILE

This motion generally occurs when the projectiles are shot straight with initial velocity, from some height  $h$  above the ground and the projectile falls downward until it hits the ground (picture 4).



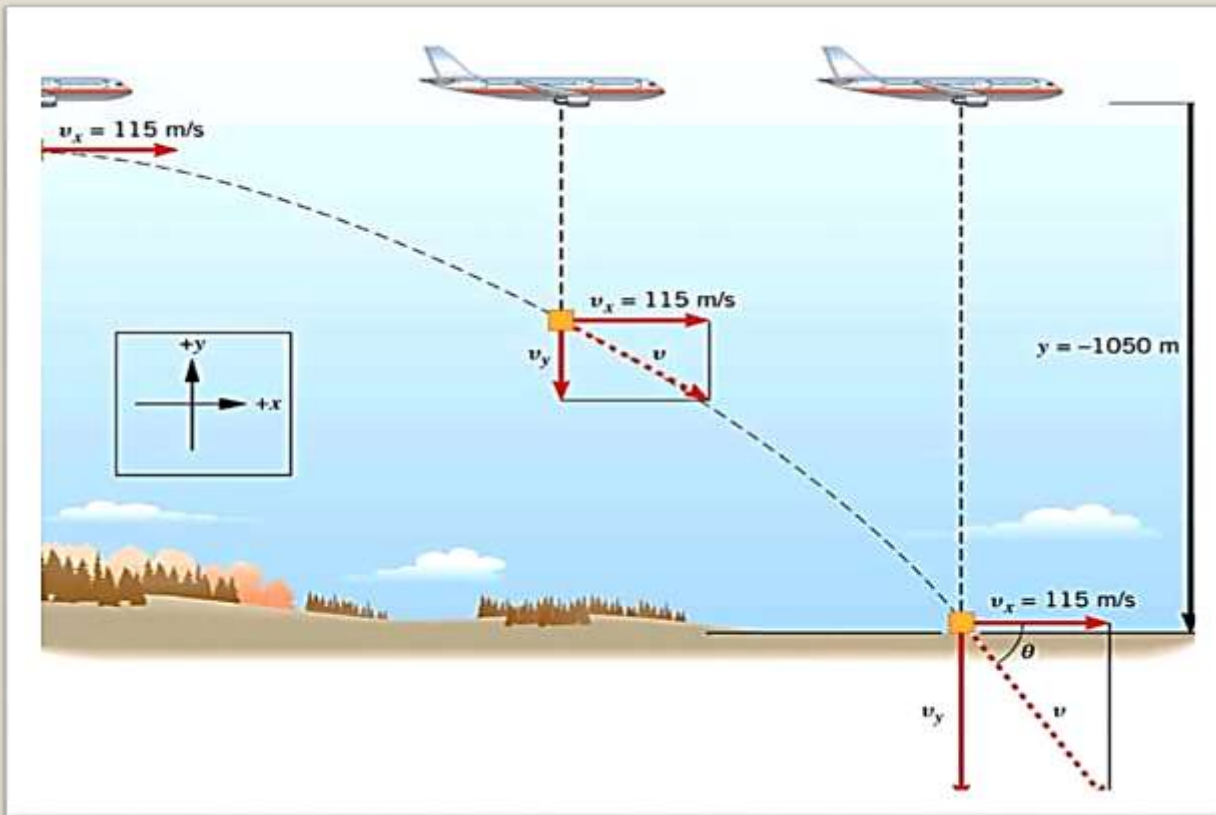
Picture 4

Cannonball shot horizontally with initial velocity 100m/a from the cliff

Experience tells us that the projectiles will follow a curved path (called **parabola**) and will eventually fall to the ground. Since the projectile is launched horizontally, the initial vertical velocity is equal to zero. Motion of the horizontally launched projectile (two dimensional motion), can be resolved into two-cases of one dimensional motion, one along the the x-axis and the other one along the y-axis. The two cases can be studied separately as two cases of one dimensional motion.

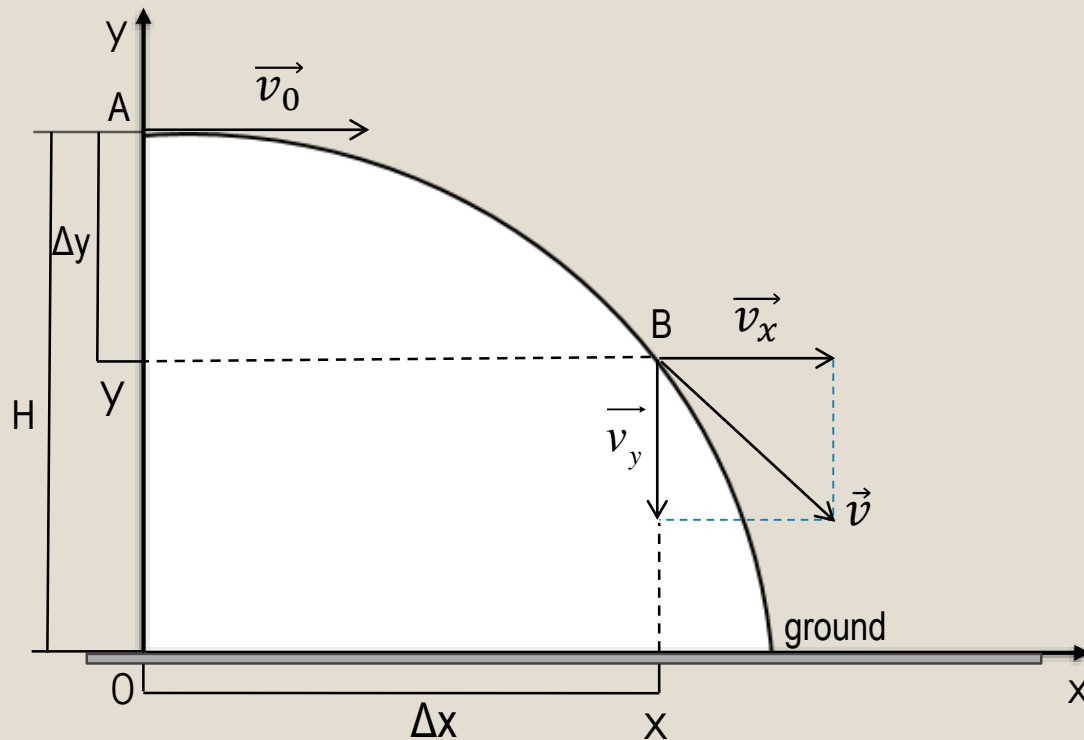
In the horizontal or x direction, moving projectile doesn't slow down in the absence of air resistance. The x-component of velocity never changes, covers equal displacements in equal time periods. This means the initial horizontal velocity equals the final horizontal velocity  $v_x = v_0$  so that means the acceleration  $a_x$  is zero (gravity does not work horizontally to increase or decrease the velocity)

In the vertical direction projectile experience effect of gravity. As a result, component of velocity is not constant, and changes (due to gravity), does not cover equal displacements in equal time periods, and component of the acceleration  $a_y = g$ . As projectile moves down the magnitude of velocity increases and the direction is downward.



Picture 5: Airplane moving horizontally with a constant velocity of 115 m/s at altitude of 1050m. The plane releases a “care package” that falls to the ground along a curved trajectory.

The easiest way to analyze what is going on is to realise that the projectile is actually undergoing two motions simultaneously—one in horizontal direction (uniform motion) and another in vertical direction (free fall). If the projectile begins motion at point A (coordinate system) at  $t=0$  and subsequently finds itself in point B, then we can say that during this time the projectile moved a horizontal displacement  $\Delta x$  and a vertical displacement  $\Delta y$ . Time taken to cover the displacement  $\Delta x$  is the same as the time taken to cover displacement  $\Delta y$ .



Picture 6: Velocity vector for curvilinear motion has a direction that is tangent to the curved line.

Displacement in horizontal direction

- $\Delta x = v_0 \cdot t$

Displacement in vertical direction

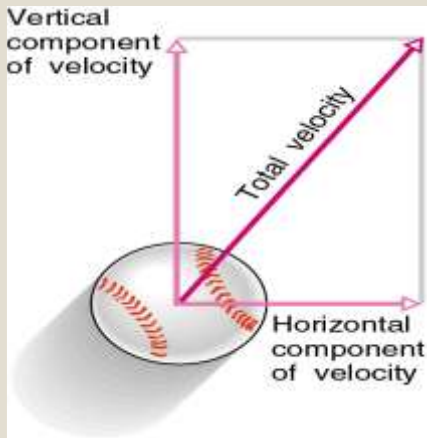
- $\Delta y = \frac{1}{2} g \cdot t^2$

x coordinate is equal to displacement in horizontal direction

- $x = v_0 \cdot t$

y coordinate is equal to height the object at a given time

- $y = H - \Delta y = H - \frac{g \cdot t^2}{2}$



Picture 7

The velocity along the x-axis  $v_x$  is constant and the velocity along the y-axis increases uniformly because of the gravitational acceleration. The total velocity (picture 7) in the direction of flight is determined using the Pythagorean theorem of the velocity components.

- $v_x$  – velocity in the horizontal direction

- $v_x = v_0$

- $v_y$  – velocity in the vertical direction

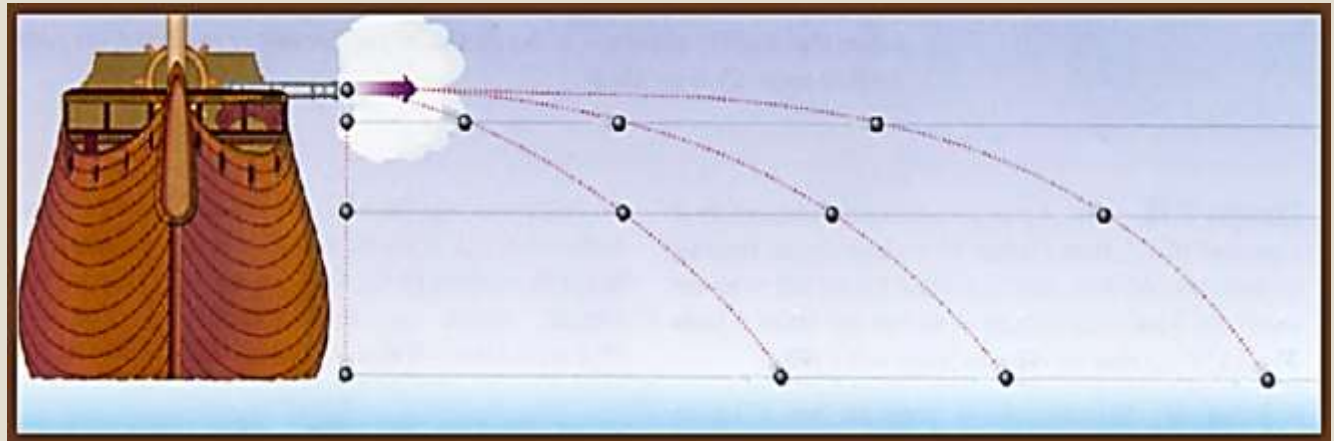
- $v_y = g \cdot t$

- Total velocity

- $$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + g^2 \cdot t^2}$$

Example:

Picture 8: Cannonballs shot horizontally with different velocities from the ship travel different distances.



But each cannonball drops the same distance in the same amount of time, since the vertical acceleration is the same for each.

## PROBLEMS

1. An object is launched horizontally from a height of 10m above the ground with initial velocity  $10 \frac{m}{s}$ .  
Calculate:
  - a) How long will it take the object to land on the ground?
  - b) How far from the table will the ball land?
  - c) With what velocity does the object hit the ground?
2. Small ball rolls horizontally from the edge of a table that is 1.30 m high. It strikes the floor at a point 1.50 m horizontally from the table edge.
  - (a) How long is the ball in the air?
  - (b) What is its velocity at the instant it leaves the table?
3. A stone is thrown horizontally. In 0.5 second after the stone began to move, the numerical value of its velocity was 1.5 times its initial velocity. Find the initial velocity of the stone.
4. A ball is launched horizontally from the edge of the cliff with initial velocity  $v_0$ . After 1s velocity makes an angle  $30^\circ$  to the horizontal. Find the initial velocity of the ball?
5. A ball is launched horizontally from the edge of the cliff with initial velocity  $15 \frac{m}{s}$ . As it hits the ground the velocity makes an angle  $60^\circ$  to the horizontal. How high is the cliff?