

# GEOMETRICAL OPTICS (RAY OPTICS)

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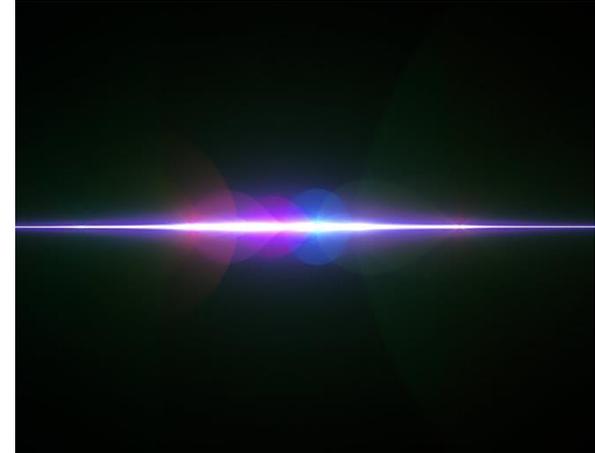
**Geometrical optics**, or **ray optics**, is a model of optics that describes light propagation in terms of rays. The ray in geometric optics is an abstraction useful for approximating the paths along which light propagates under certain circumstances.

Geometrical optics is based on four laws:

- ◆ the law of rectilinear propagation of light
- ◆ the law of independence of light rays
- ◆ the law of reflection
- ◆ the law of refraction of light.

The **law of rectilinear propagation of light** states that light propagates in straight lines in homogeneous media (picture 1).

The **law of independence of light rays** states that rays do not perturb each other upon intersection.



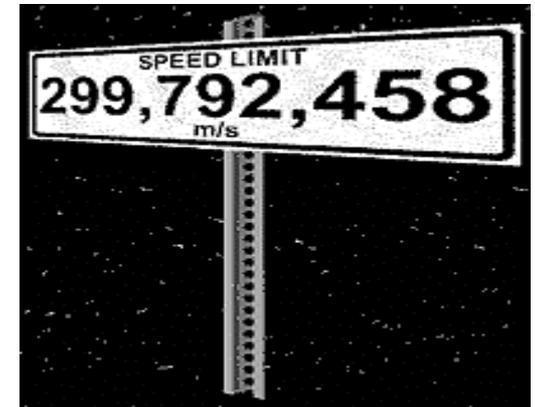
Picture 1

## THE SPEED OF LIGHT AND THE INDEX OF REFRACTION

In vacuum the speed of light is:  $c = 2.99792458 \times 10^8$  m/s

When light passes from one transparent medium to another, it's refracted because the speed of light is different in the two media. The **index of refraction**,  $n$ , of a medium is defined as the ratio

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}$$

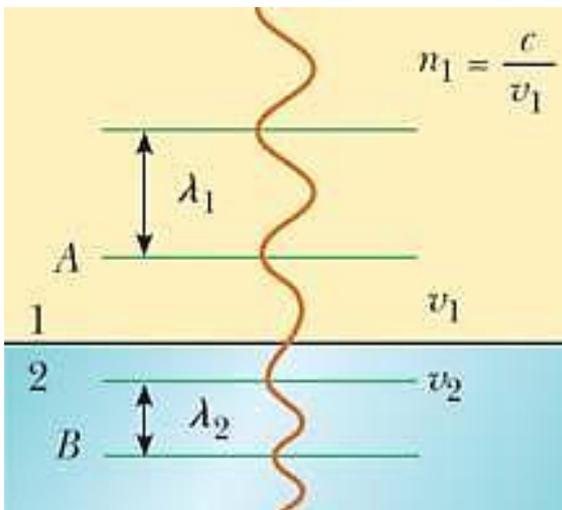


Picture 2

From this definition, we see that the index of refraction is a dimensionless number that is greater than or equal to 1 because  $v$  is always less than  $c$ . Further,  $n$  is equal to one for vacuum.

Material	Index of Refraction (n)
Vacuum	1.000
Air	1.000277
Water	1.333333
Ice	1.31
Glass	About 1.5
Diamond	2.417

**As light travels from one medium to another, its frequency doesn't change.**



Picture 3

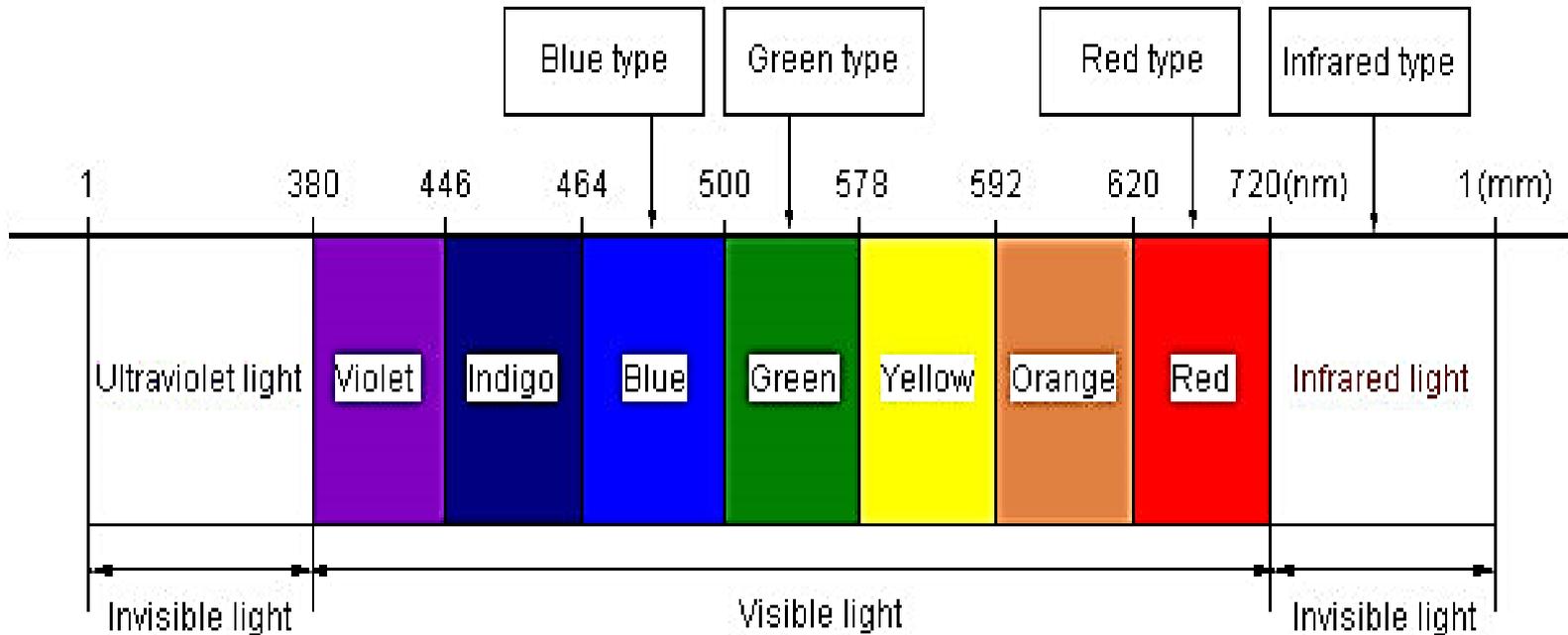
As the wave moves from medium 1 to medium 2, its wavelength changes, but its frequency remains constant.

Because the relation  $v = f \cdot \lambda$  must be valid in both media and because  $f_1 = f_2 = f$  we see that  $v_1 = f \cdot \lambda_1$  and  $v_2 = f \cdot \lambda_2$

A relationship between the index of refraction and the wavelength can be obtained by dividing these two equations and making use of the definition of the index of refraction:  $\lambda_1 n_1 = \lambda_2 n_2$

Picture 3 is a schematic representation of this reduction in wavelength when light passes from medium 1 (vacuum) into a transparent medium 2.

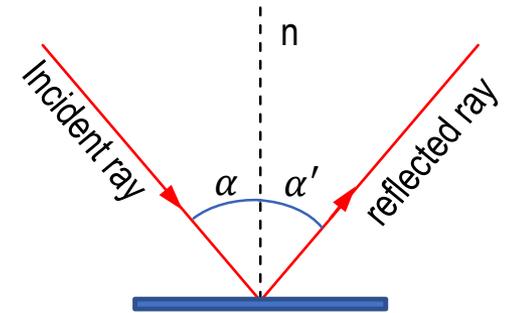
The word light usually refers to **visible light**, which is the visible spectrum that is visible to the human eye and is responsible for the sense of sight. Visible light is usually defined as having wavelengths in the range of 400–700 nanometres (nm), or  $4.00 \times 10^{-7}$  to  $7.00 \times 10^{-7}$  m (picture 4) between the infrared (with longer wavelengths) and the ultraviolet (with shorter wavelengths). This wavelength means a frequency range of roughly 430–750 terahertz (THz).



picture 4

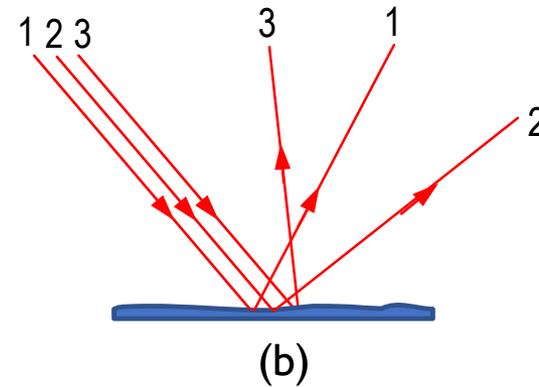
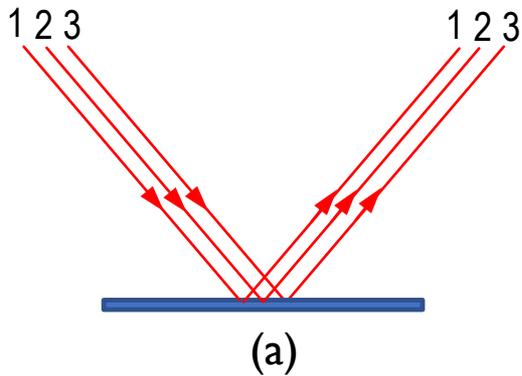
# LAW OF REFLECTION OF LIGHT

The **law of reflection of light** states that the reflected ray lies in the same plane with the incident ray and with the normal to the reflecting surface at the point of incidence (picture 5), the angle of reflection being equal to the angle of incidence. The angle of incidence is the angle  $\alpha$  between the normal and the incident ray, while the angle of reflection is the angle  $\alpha'$  between the normal and the reflected ray.



Picture 5

If the beam of parallel rays falls on a flat smooth surface, a parallel beam will be obtained in the reflection (picture 6-a). If a surface is not smooth for given rays, diffuse reflection is observed since rays incident on such a surface are reflected in all directions (picture 6-b).



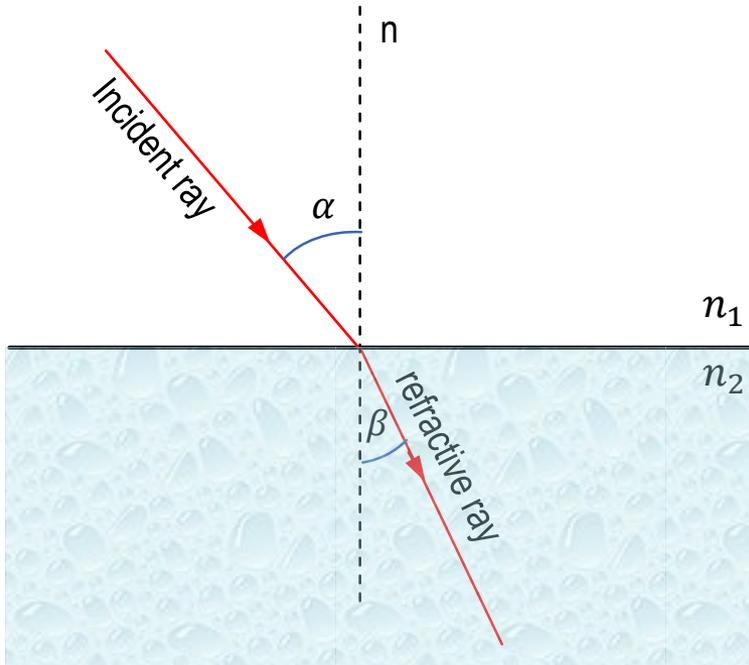
Picture 6

# THE LAW OF REFRACTION

The law of refraction, which is generally known as Snell's law, governs the behaviour of light-rays as they propagate across a sharp interface between two transparent medium.

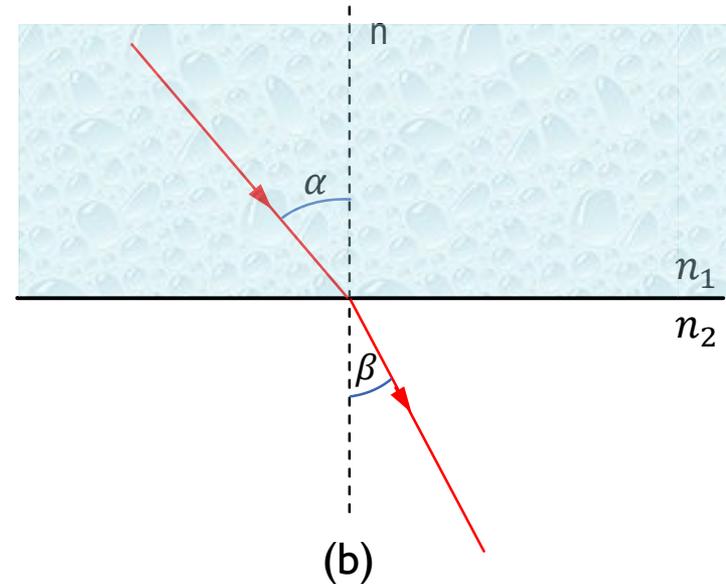
**Laws of refraction of light** state that the ratio of the sines of the angle of incidence  $\alpha$  and of the angle of refraction  $\beta$  is equal to the ratio of their absolute refractive index:


$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$



(a)

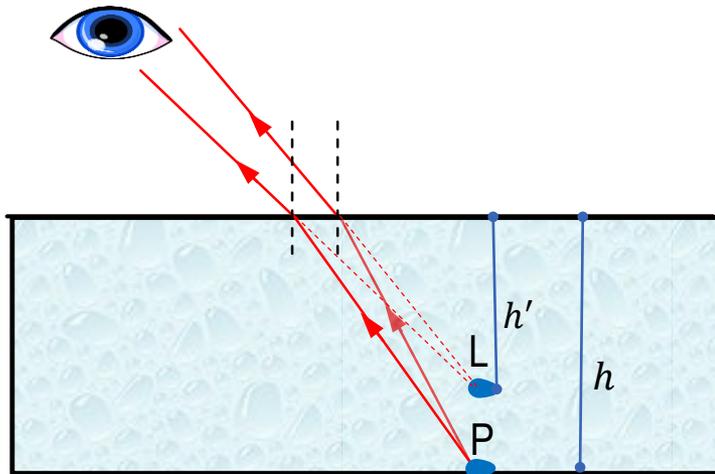
The quantities  $n_1$  and  $n_2$  are termed the refractive indices of media 1 and 2, respectively. Thus, the law of refraction predicts that a light-ray always deviates more towards the normal in the optically denser medium: i.e., the medium with the higher refractive index. (picture 7-a)



Picture 7

Also the law of refraction predicts that a light-ray always deviates more away from the normal in the optically rarer medium: i.e., the medium with the rarer refractive index. (picture 7-b)

## APPARENT DEPTH

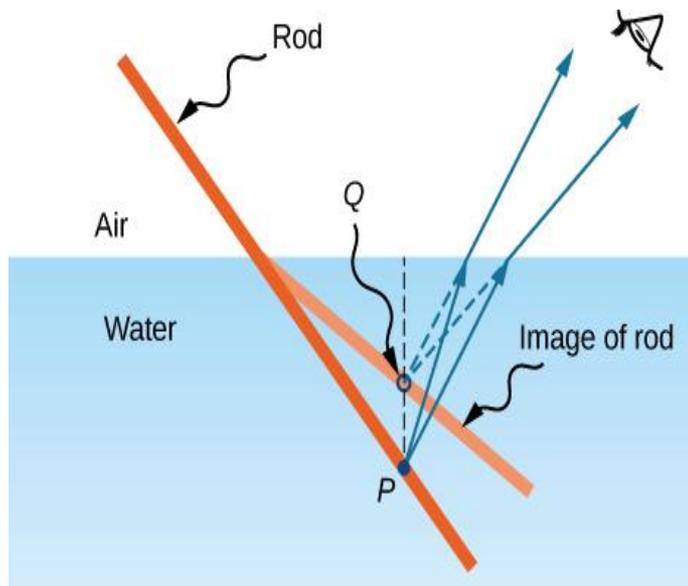


Picture 7

An object placed in a denser medium, when viewed from rarer medium appears to be at a lesser **depth** than its real **depth** due to refraction of light.

Picture 7 shows two rays passing from an object P in water. Moving into the air, they refract and refracted rays come to the eye. The extenders of these rays are cut at point L. So if the object is at a depth h in the water, an observer from the air sees his character at a lower depth h'. It is shown when the observer looks in the direction of the normal to the surface of the water (or at a smaller angle relative to the normal), the real and apparent depths are related by the equation:

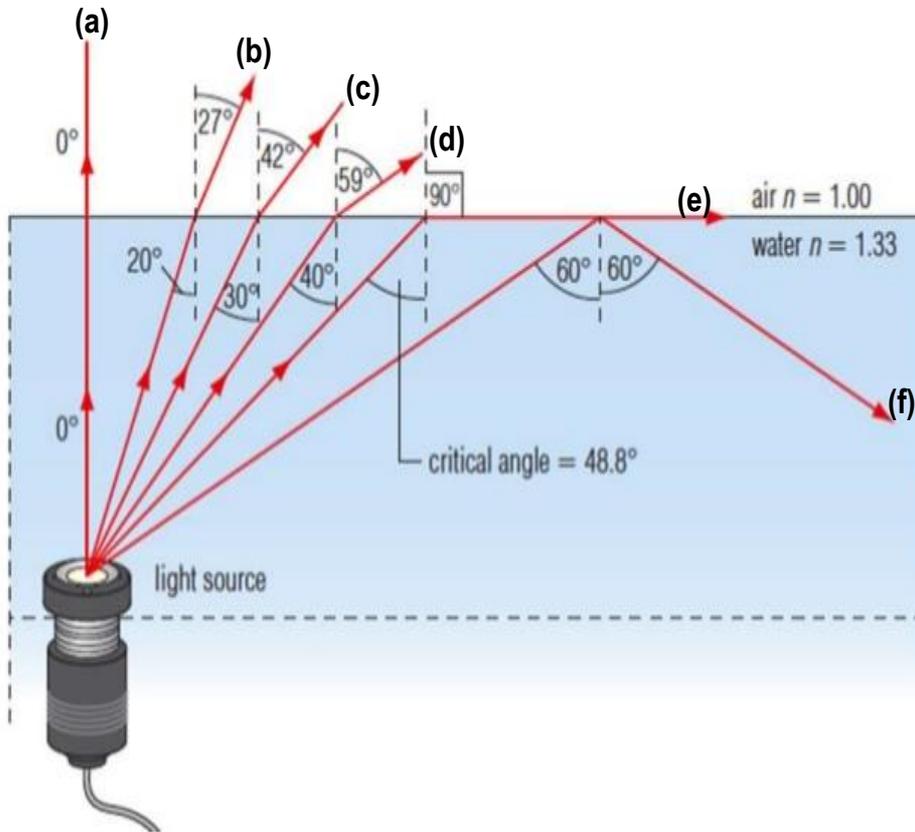
$$\frac{h}{h'} = n$$



Picture 8

If we look at a straight rod partially submerged in water, it appears to bend at the surface (picture 8). The reason behind this curious effect is that the image of the rod inside the water forms a little closer to the surface than the actual position of the rod, so it does not line up with the part of the rod that is above the water. The same phenomenon explains why a fish in water appears to be closer to the surface than it actually is.

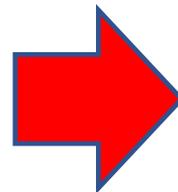
# TOTAL INTERNAL REFLECTION



Picture 9

Picture 9 shows rays of monochromatic light from a light source in water incident on the interface between the water and air. For ray *a*, which is perpendicular to the interface, part of the light reflects at the interface and the rest travels through it with no change in direction. For rays *b* through *d*, which have progressively larger angles of incidence at the interface, there are also both reflection and refraction at the interface. As the angle of incidence increases, the angle of refraction increases; for ray *e* it is  $90^\circ$ , which means that the refracted ray points directly along the interface. The angle of incidence giving this situation is called the **critical angle**  $\alpha_c$ . For angles of incidence larger than  $\alpha_c$ , such as for ray *f*, there is no refracted ray and *all* the light is reflected; this effect is called **total internal reflection**.

To find  $\alpha_c$ , we arbitrarily associate subscript 1 with the water and subscript 2 with the air, and then we substitute  $\alpha_c$  for  $\alpha_1$  and  $90^\circ$  for  $\beta$ .

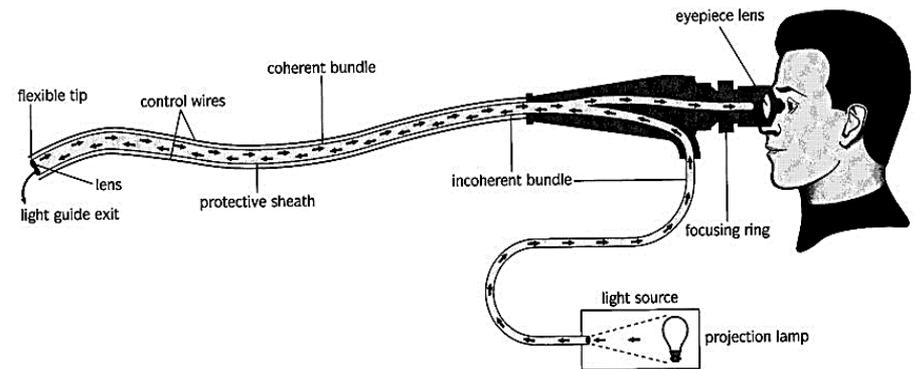


$$\alpha_c = \arcsin \frac{n_2}{n_1}$$

• **critical angles of different materials**

Medium	Refractive index	Critical angle
Glass	1.50–1.70	30°–42°
Water	1.33	49°
Perspex	1.5	42°
Diamond	2.42	24°

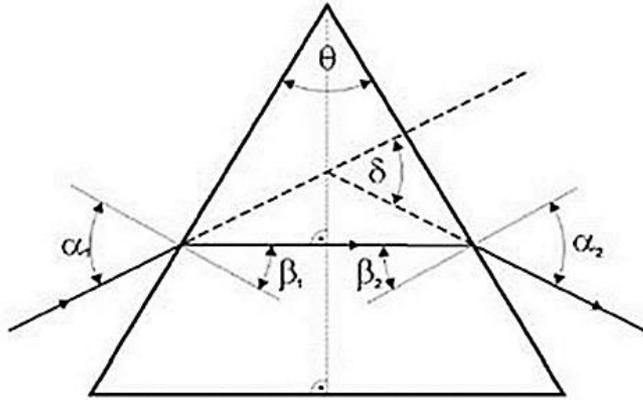
Total internal reflection has found many applications in medical technology (picture 10). For example, a physician can view the interior of an artery of a patient by running two thin bundles of optical fibers through the chest wall and into an artery. Light introduced at the outer end of one bundle undergoes repeated total internal reflection within the fibers so that, even though the bundle provides a curved path, most of the light ends up exiting the other end and illuminating the interior of the artery. Some of the light reflected from the interior then comes back up the second bundle in a similar way, to be detected and converted to an image on a monitor's screen for the physician to view.



An endoscope uses total internal reflection to enable a doctor to look deep inside the body. It enables key hole surgery to take place.

Picture 10

## DISPERSION AND PRISMS



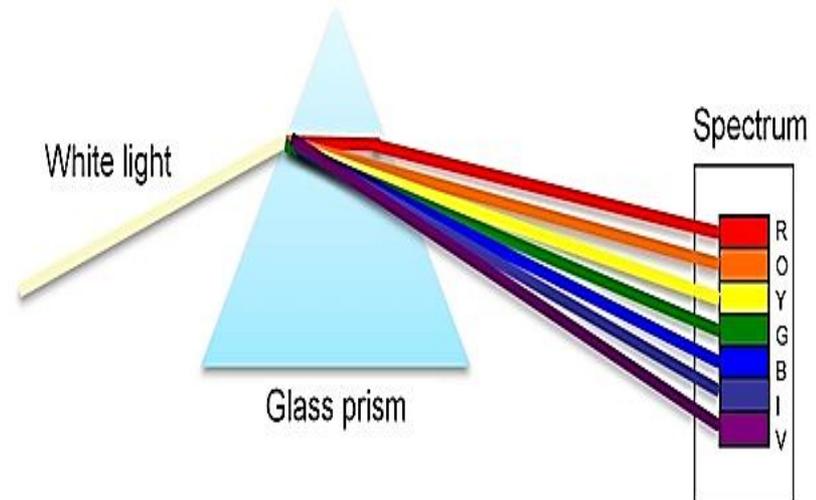
Picture 11

The dependence of the index of refraction on wavelength is called **dispersion**. Because  $n$  is a function of wavelength, Snell's law indicates that **the angle of refraction made when light enters a material depends on the wavelength of the light**.

The index of refraction for a material usually decreases with increasing wavelength. This means that violet light ( $\lambda=400$  nm) refracts more than red light ( $\lambda=650$  nm) when passing from air into a material. To understand the effects of dispersion on light, consider what happens when light strikes a prism (picture 11). A ray of light of a single wavelength that is incident on the prism from the left emerges bent away from its original direction of travel by an angle  $\delta$ , called the **angle of deviation**.

Angle of deviation is increased with index of refraction. Now suppose a beam of white light (a combination of all visible wavelengths) is incident on a glass prism (picture 12).

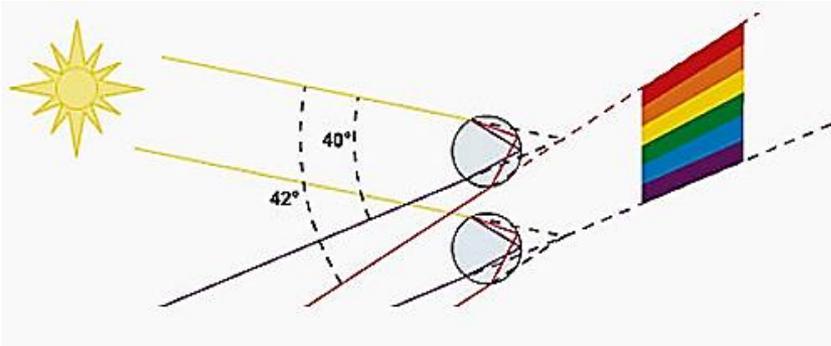
The glass prism split the light into a band of seven colours on his wall. This band of colours represent 'spectrum'. The order of colours from the lower end of spetrum is violet (V), indigo (I), blue (B), green (G), yellow (Y), orange (O), and red (R).



Picture 12

# THE RAINBOW

The dispersion of light into a spectrum is demonstrated most vividly in nature through the formation of a rainbow, often seen by an observer positioned between the Sun and a rain shower.



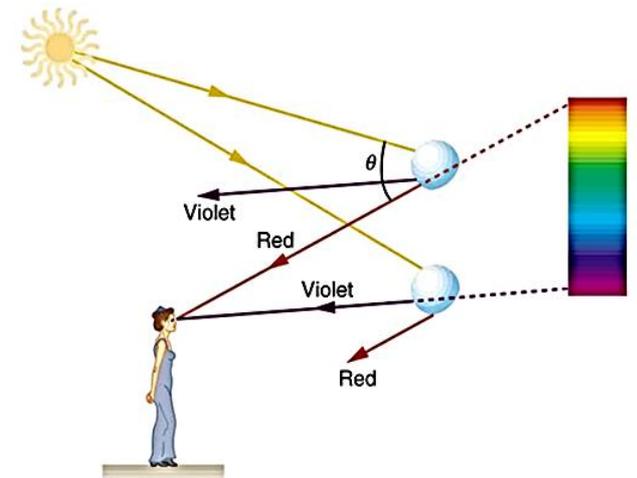
Picture 13

Refraction of sunlight by a spherical raindrop.

To understand how a rainbow is formed, consider picture 13. A ray of light passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows: It is first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air. The rays leave the drop so that the angle between the incident white light and the returning violet ray is  $40^\circ$  and the angle between the white light and the returning red ray is  $42^\circ$ .

Now consider an observer viewing a rainbow, as in picture 14.

If a raindrop high in the sky is being observed, the red light returning from the drop can reach the observer because it is deviated the most, but the violet light passes over the observer because it is deviated the least. Hence, the observer sees this drop as being red. Similarly, a drop lower in the sky would direct violet light toward the observer and appear to be violet. (The red light from this drop would strike the ground and not be seen.) The remaining colors of the spectrum would reach the observer from raindrops lying between these two extreme positions.



Picture 14

## PROBLEMS

1. A light ray passes from air to water at an angle of incidence  $\alpha = 60^\circ$

- Find the angle of refraction?
- Find the speed of light in water?

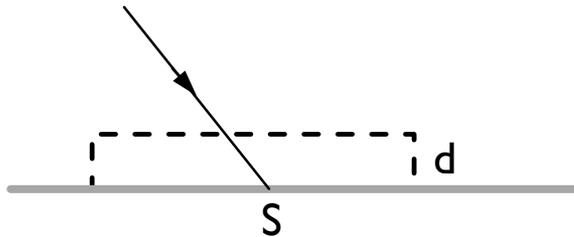
The refractive index of the water is 1.33.

2. Light of wavelength 589 nm in vacuum passes through a piece of fused quartz of index of refraction  $n = 1.46$ .

- Find the speed of light in fused quartz?
- What is the wavelength of this light in fused quartz?
- What is the frequency of the light in fused quartz?

3. A 1.8 m long vertical pole extends from the bottom of swimming pool to the point 30 cm above the water. Sunlight is incident at  $30^\circ$  above the horizontal. What is the length of the shadow of the pole on the bottom level of the pool?

4. There is a sheet of paper on the table. A ray of light strikes it at an angle of incidence of  $30^\circ$  forming the bright spot S. How much will the bright spot be moved if a  $d = 5$  cm thick glass slab is placed on the paper as shown in the picture. The refractive index of the glass is 1.5.



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5. A monochromatic ray falls onto the side surface of an equilateral glass prism, and after refraction travels parallel to its base. (In this case the angle of incidence  $\alpha$  is chosen so that the emerging ray also makes the same angle  $\alpha$  with the normal to the other face). When the ray emerges from the prism, it is deflected by an angle  $\delta$  from its initial direction. Determine the relationship between the refraction angle of the prism  $\theta$ , the deflection of the ray  $\delta$  and the refractive index  $n$ .
6. A ray of light emerges from turpentine into air. The limit angle of total internal reflection for this ray is  $42^\circ$ . What is the propagation speed of light in turpentine?
7. A point source of light is placed on the bottom of a vessel filled with water to a height of 3m. A circular opaque plate so floats on the surface of the water that its centre is above the source of light. What should the minimum radius of this plate be to prevent all the rays from emerging through the water surface?