# WAVE OPTICS

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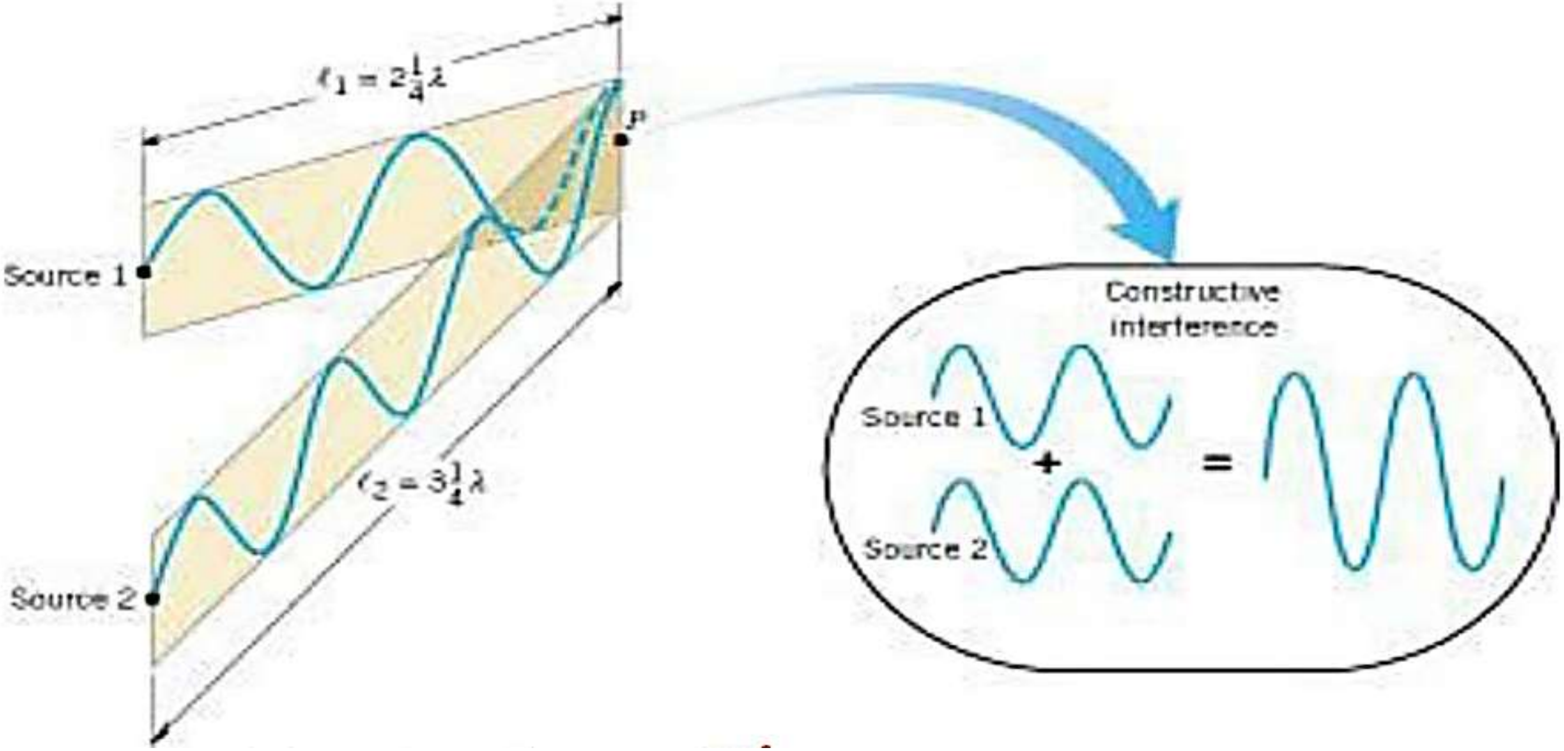
In physics, **physical optics**, or **wave optics**, is the branch of optics that studies interference (picture 1), diffraction, polarization, and other phenomena for which the ray approximation of geometric optics is not valid. Two theories of light, viz. the Newton *corpuscular theory* and the Huygens *wave theory,* were put forward almost simultaneously at the end of the 17th century.
According to the corpuscular theory formulated by Newton luminous bodies emit tiniest particles (corpuscles) propagating along straight lines in all directions and causing a visual perception in an eye.
According to the wave theory, a luminous body causes elastic oscillations in a special medium (ether) filling the entire space. These oscillations propagate in the ether like sonic waves in air. At the time of Newton and Huygens, most scientists supported the Newton corpuscular theory which explained fairly well all optical phenomena known at that time. At the beginning of the 19th century, the Huygens wave theory, which was refuted by contemporaries, was developed and perfected by Young and Fresnel and received a wide recognition.

Picture 1: The blue of the top surface of a *Morpho* butterfly wing is due to optical interference and shifts in color as your viewing perspective changes.

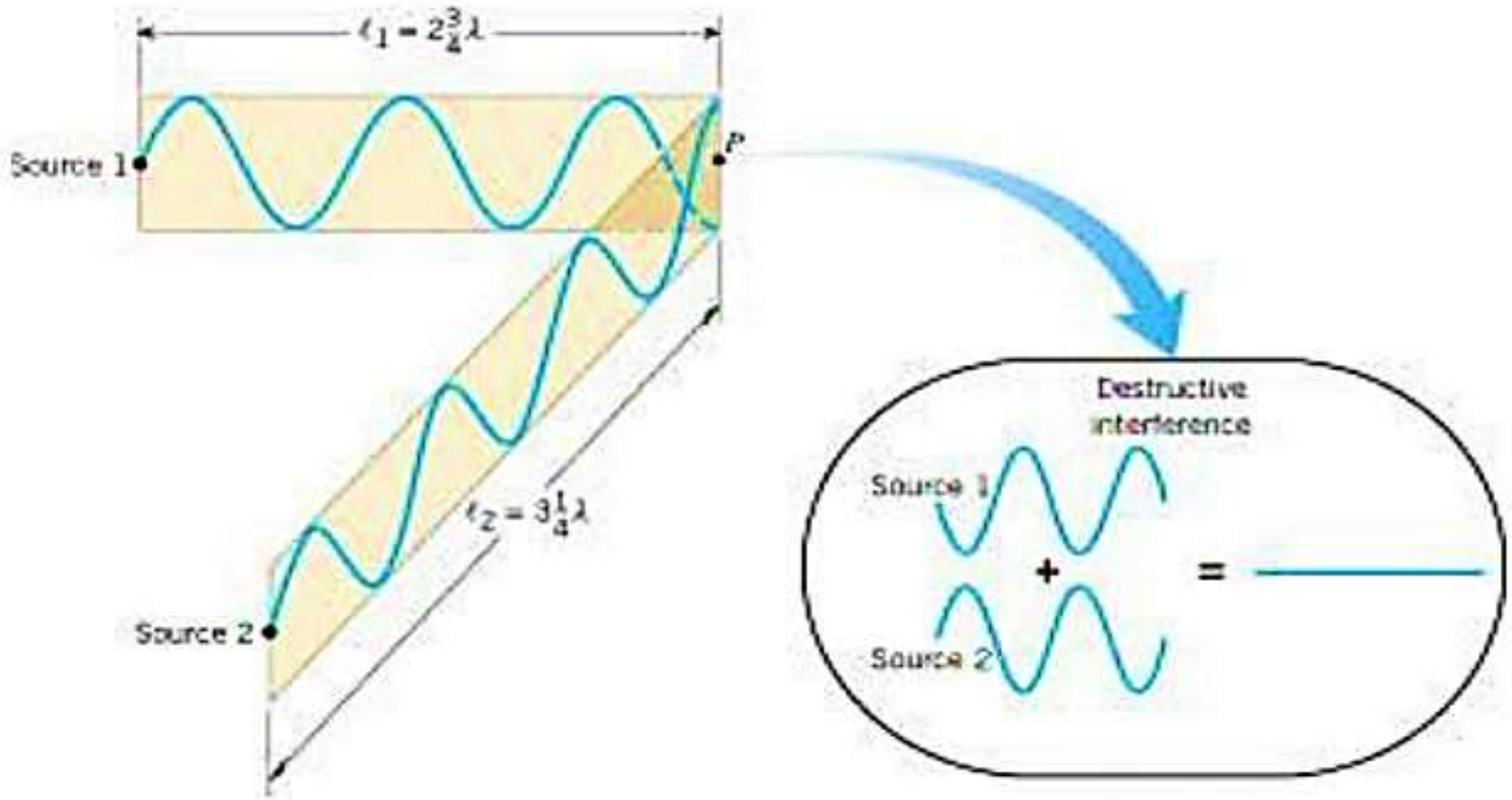
# INTERFERENTION OF LIGHT

**Interference of light wave is the phenomena in which two light waves from coherent sources are superimposed to give dark and bright fringes.**




Picture 2

The waves arriving at P from the two conherent sources travel the different distances.(picture 2)
If the amplitudes of two waves have the same sign (either both positive or both negative), they will add together to form a wave with a larger amplitude. Point P is at distant $x_1$ = 2.25λ from point source $S_1$ and for $x_2$ = 3.25λ from point source $S_2$ . If at point P the path difference is either zero or some integral multiple of the wavelength, the two waves are in phase at *P* and constructive interference results.




Picture 3

If the two amplitudes have opposite signs, they will subtract to form a combined wave with a lower amplitude (picture 3). If at point P the path difference is half- integral multiple of the wavelength, the two waves are in counterphase at *P* and destructive interference results.

Interference effects in light waves are not easy to observe because of the short wavelengths involved (about 400-750 nm). For sustained interference between two conherent sources of light to be observed, the sources must contain a constant phase with respect to each other , and must have identical frequency.

## ● Conditions for interference ●

The monochromatic light waves that extends in the direction of the x-axis is written in form:

$E = A\cos(\omega t - kx)$ → $E$ is the intensity of the electric field strenght vector, A is the amplitude of the wave, $k = 2\pi / \lambda$ is the wave number, λ is wavelength and $\omega = 2\pi\nu$ is the angular frequency.

Let us assume that two waves of the same frequency, being superposed on each other, produce oscillations of the same direction. There equations are given by: →

$$E_1 = A_1\cos(\omega t - ks_1)$$
$$E_2 = A_2\cos(\omega t - ks_2)$$

Resultant vector at a certain point in space is given by: →

$$E_1 = A_1\cos(\omega t - ks_1)$$
$$E_2 = A_2\cos(\omega t - ks_2)$$

→ $$E = E_1 + E_2 = A\cos(\omega t - \varphi)$$

Resultant vector has the same angular frequency ω, while the amplitude A and phase $\varphi$ are determinated by expresion: $A_1\cos(\omega t - ks_1) + A_1\cos(\omega t - ks_1) = A\cos(\omega t - \varphi)$

The intensity of the wave is directly proportional to the square of its amplitude, . I$\sim A^2$
After simple mathematical transformations, the intensity of the resulting wave is given by:
I$\sim A^2$=$A_1^2$+ $A_2^2$+2$A_1 A_2$cos(k$\Delta s$) $\rightarrow I = I_1 + I_2$+2$\sqrt{I_1 I_2}$cos(k$\Delta$s), where $\Delta s = s_2 - s_1$ is path difference of waves and $I = I_1 + I_2$+2$\sqrt{I_1 I_2}$cos($\Delta\varphi$)$\cdots (*)$, where $\Delta\varphi = \varphi_2 - \varphi_1$ is phase difference of the waves.

**1.** The phase difference $\delta$ for incoherent waves varies continuously and takes on any values with an equal probability
Hence, the time-averaged value of cos $\delta$ equals zero. Therefore $I = I_1 + I_2$

**2.** For conherent waves, cos$\delta$ has a time-constant value, so that and $I = I_1 + I_2$+2$\sqrt{I_1 I_2}$cos($\Delta\varphi$)

- At the points of space for which cos$\Delta\varphi > o$, the intensity *I* will exceed $I_1 + I_2$ .
- At the points for which cos $\Delta\varphi < 0$, it will be smaller than $I_1 + I_2$
- As a result, maxima of the intensity will appear at some spots and minima at others.

Interference manifests itself especially clearly when the intensity of both interfering waves is the same: $I_1 = I_2 = I_0$

$$\boldsymbol{I} = \begin{pmatrix} \mathbf{4} \cdot \boldsymbol{I_0} \quad \boldsymbol{for} \ \mathbf{cos}(\boldsymbol{\varphi_2} - \boldsymbol{\varphi_1}) = \mathbf{1} \\ \mathbf{0} \quad \boldsymbol{for} \ \mathbf{cos}(\boldsymbol{\varphi_2} - \boldsymbol{\varphi_1}) = -\mathbf{1} \end{pmatrix}$$

Constructive or destructive interference occurs in those points to which the waves come with constant time-value phase

$$\boldsymbol{\varphi_2} - \boldsymbol{\varphi_1} = \begin{pmatrix} \mathbf{2}\boldsymbol{m\pi} \quad - \boldsymbol{constructive\ interference} \\ (\mathbf{2}\boldsymbol{m} + \mathbf{1})\boldsymbol{\pi} \quad - \boldsymbol{destructive\ interference} \end{pmatrix}$$

**Conditions for constructive interference:**

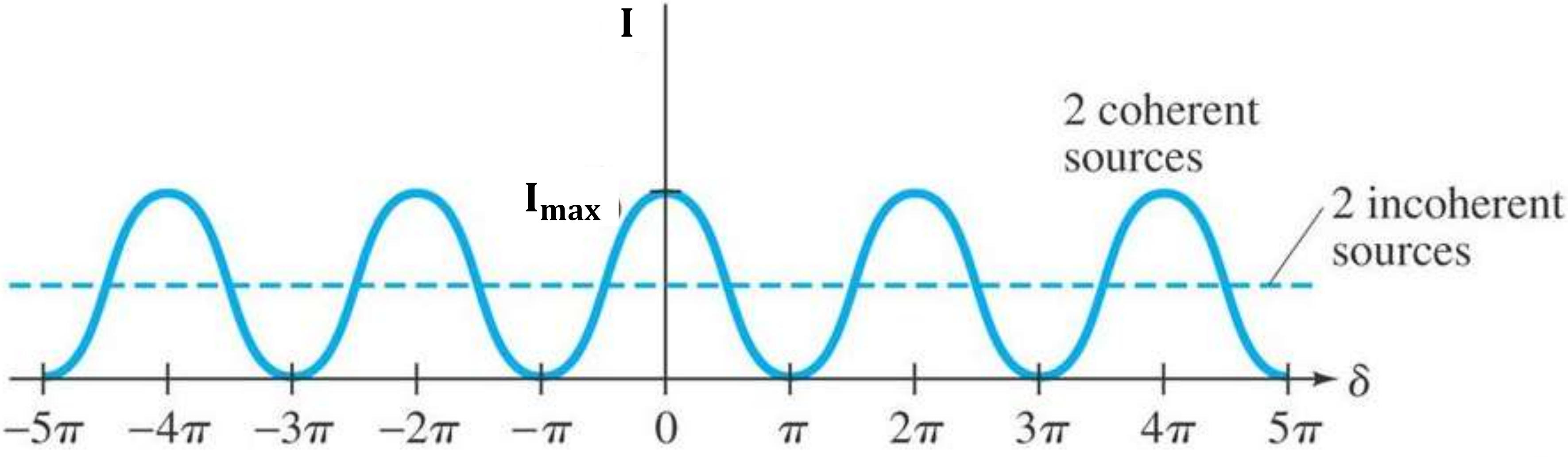
$\Delta s = m \cdot \lambda$ $(\mathrm{m} = 0, \pm 1, \pm 2 \cdots)$

**Conditions for destructive intererence:**

: $\Delta s = m \cdot \lambda + \frac{\lambda}{2}$ $(\mathrm{m} = 0, \pm 1, \pm 2 \cdots)$

$I_{max} \sim (A_1 + A_2)^2$ or $I_{max} > I_1 + I_2$

$I_{min} \sim (A_2 - A_1)^2$ tj $I_{min} < I_1 + I_2$




Picture 4: Distribution of the intensity as function of phase difference Δφ

## ● Optical path length (OPL) ●

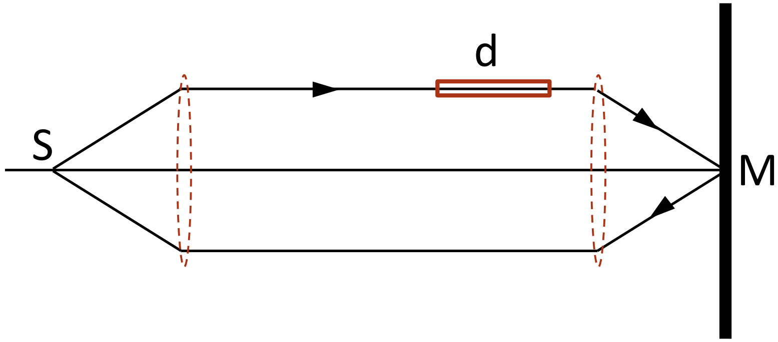
If the light extends through the medium of the refractive index $n$> 1, then its wavelength changes. Then instead of the geometric length of path (s), the corresponding optical path length ($s'$) is the product of the geometric length of the path followed by light through a given medium, and the index of refraction of the medium through which it propagates: $s'=n \cdot s$.

**The optical path difference can be calculated using equation:**

$$\Delta \boldsymbol{s} = \boldsymbol{n_2 s_2} - \boldsymbol{n_1 s_1}$$

where $n_1$ and $n_2$ the refraction indexs of the medium 1 and medium 2.

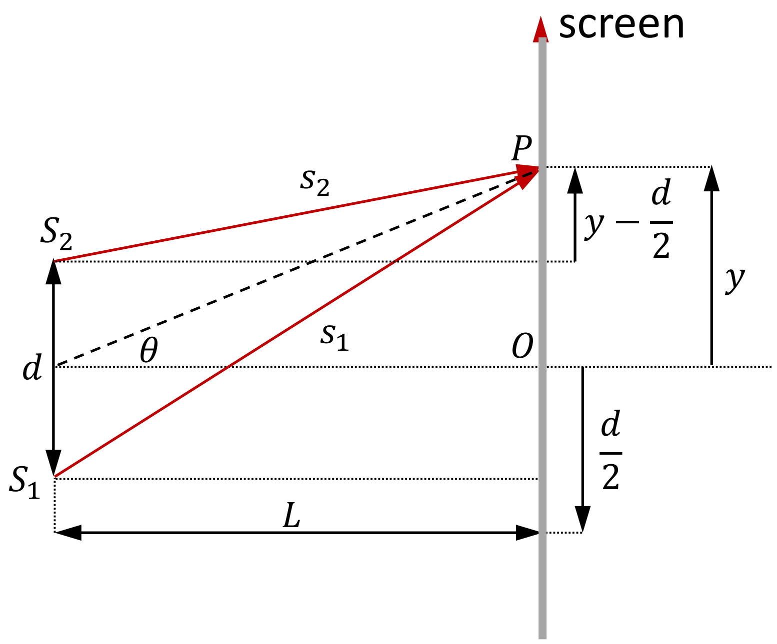
Let the light source S be placed in the focus of covex lens (picture 5). One ray travel the distance d for example through glass, and the other ray travel the same distance d through the air of the refractive index $n$≈1.
The optical path difference for these two rays is:
$\Delta s = nd - d = d\,(n-1)$.

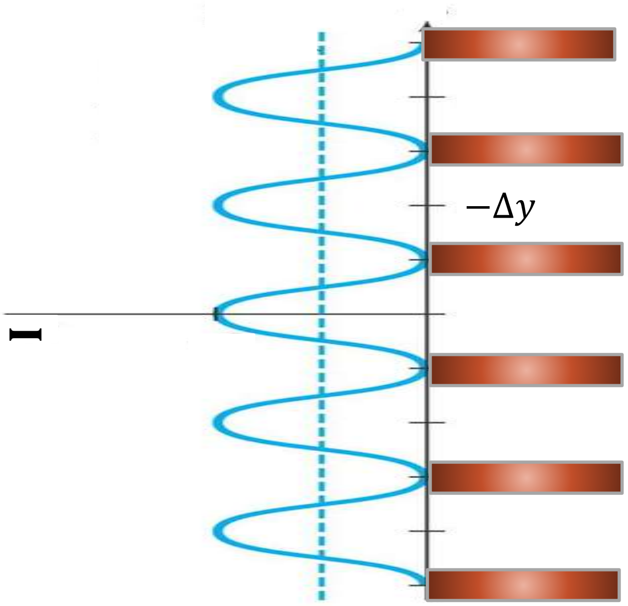



Picture 5

## ► YOUNG EXPERIMENT ◄

Ttwo monochromatic coherent light sources $S_1$ and $S_2$ be at a distance d from each other (picture 6). The distance from the source to the screen is $L»d$. An arbitrary point P is displayed on the screen whose position is determined by the coordinate x from the center of screen(point O). We shall choose the beginning of our readings at point *0* relative to which $S_1$ and $S_2$ are arranged symmetrically. If we introduce a screen into the interference field, we shall see on it an interference pattern having the form of alternating light and dark fringes.




Picture 6

Calculations shows that: $s_1^2 = L^2 + \left(y + \frac{d}{2}\right)^2$ $s_2^2 = L^2 + \left(y - \frac{d}{2}\right)^2$ $s_1^2 - s_2^2 = (s_1 + s_2)(s_1 - s_2) = 2yd$

$s_1 + s_2 \approx 2L$, and $s_1 - s_2 = \Delta s$. Calculations shows that optical path distance is: $\Delta s = \frac{yd}{L}$

For point P the introduction of value $\Delta s$ into condition for conherent interference shows that intensity maxima will be observed at values of $y_m$ equal to:

$$\Delta s = m \cdot \lambda = \frac{y_m d}{L} \Rightarrow y_m = m\frac{L}{d}\lambda \Rightarrow \text{where: } m = 0,1,2 \cdots$$

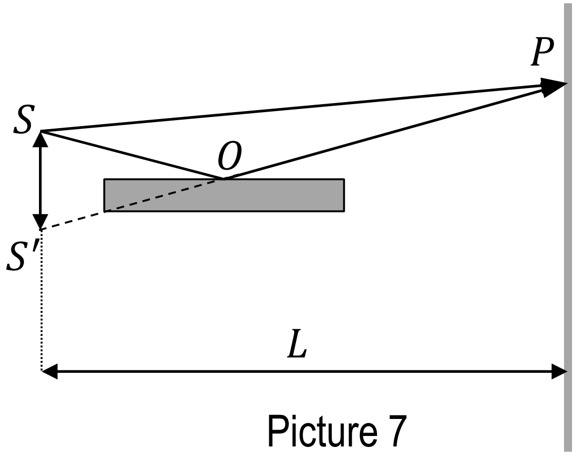
For point P the introduction of value $\Delta s$ into condition for destructive interference shows that intensity minima will be observed at values of $y_m$ equal to:

$$y_m = \frac{2m+1}{2}\frac{L}{d}\lambda \Rightarrow \text{where: } m = 0,1,2 \cdots$$

The distance between two adjacent intensity maxima (the distance between interference fringes), and the distance between adjacent intensity minima (the width of an interference fringe) is given by:

$$\Delta y = y_m - y_{m-1} = \frac{L}{d}\lambda$$

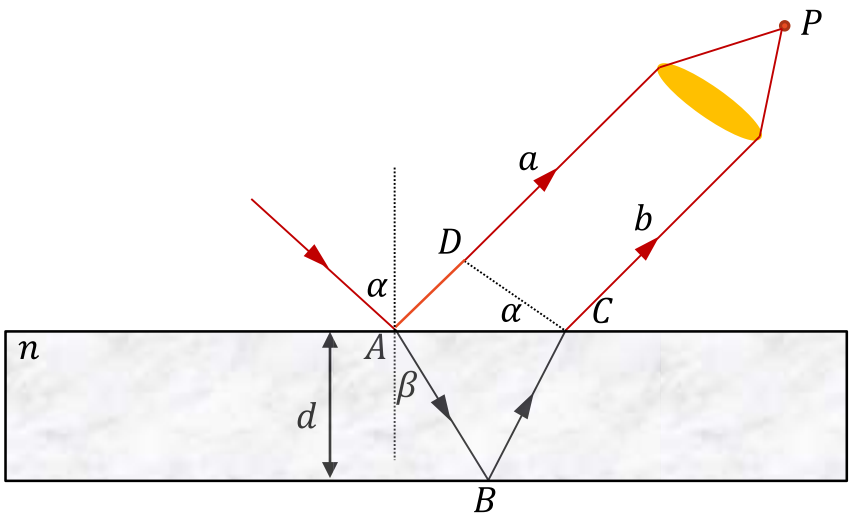
**● Lloyd's mirror ●**




Picture 7

Another simple, yet ingenious, arrangement for producing an interference pattern with a single light source is known as *Lloyd's mirror*. A point source of light is placed at point *S*, close to a mirror, as illustrated in picture 7. Light waves can reach the viewing point *P* either by the direct path *SP* or by the path involving reflection from the mirror.
In general, an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has an index of refraction higher than the one in which the wave was traveling.

# ● Interference of light reflected from thin films ●




Picture 8

When a light wave falls on a thin transparent film, reflection occurs from both surfaces of the plate. The result is the production of two light waves that in known conditions can interfere. Assume that a light wave falls on a transparent plane-parallel film (Picture 8).

The film reflects upward two parallel beams of light. One of them was formed as a result of reflection from the top surface of the film and the second as a result of reflection from its bottom surface.

Light wave undergoes a phase change of 180° upon reflection from a medium that has an index of refraction higher than the one .Because of this we need to add (or subtrack) half of wavelength to $\Delta s$ .

The geometric path difference of reflected waves is:

$$\Delta s = \overline{AB} + \overline{BC} - \overline{AD} \pm \frac{\lambda}{2}$$

The optical path difference of reflected waves is :

$$\Delta s = n(\overline{AB} + \overline{BC}) - \overline{AD} \pm \frac{\lambda}{2}$$

$$n(\overline{AB} + \overline{BC}) = \frac{2nd}{cos\beta}$$

$\overline{AC} = 2d\tan\beta$, and $\overline{AD} = 2d\tan\beta\sin\alpha$. The optical path difference is:

$$\Delta s = \frac{2nd}{\cos\beta} - 2d\tan\beta\sin\alpha - \frac{\lambda}{2} \Rightarrow \begin{matrix} \sin\beta = \frac{1}{n}\sin\alpha \\ \cos\beta = \sqrt{1 - \frac{1}{n^2}\sin\alpha^2} \end{matrix} \Rightarrow \Delta s = 2d\sqrt{n^2 - \sin\alpha^2} - \frac{\lambda}{2}$$

In point P conditions for constructive interference is:

$$2d\sqrt{n^2 - \sin\alpha^2} - \frac{\lambda}{2} = m\lambda \Rightarrow 2d\sqrt{n^2 - \sin\alpha^2} = (2m+1)\frac{\lambda}{2}, \text{ m= } 0,1,2$$

In point P conditions for destructive interference is:

$$2d\sqrt{n^2 - \sin\alpha^2} = m\lambda, \text{ m= } 0,1,2$$

Example:Soap bubbles on water.
The colors are due to interference between light rays reflected from the front and back of the thin film of soap making up the bubble. The color depends on the thickness of the film, ranging from black where the film is at its thinnest to magenta where it is thickest.

## PROBLEMS

1. A screen is separated from a double light sources by 1.20 m. The distance between the two sources is 0.030 mm. The second-order bright fringe is measured to be 4.50 cm from the centre. Determine:
   (a) the wavelength of the light (b) the distance between adjacent bright fringes?
2. In a Young experiment, the distance between light sources is 5.0 mm and the sources are 1.0 m from the screen. Two interference patterns can be seen on the screen: one due to light of wavelength 480nm, and the other due to light of wavelength 600 nm. What is the separation on the screen between the third-order bright fringes?
3. In Young experiment, if the distance between light sources reduced 1,5 times, what happens to the distance between adjacent bright fringes?
4. Find the minimum thickness of a thin soap film with refractive index 1,33 at which light with wavelength 520nm experiences maximum reflection above the layer of soap? The incidence angle of light is equal to 30°.
5. A thin layer of oil (d=1,8$\mu m$) floats on a puddle of water. Find the incident angle at which light with wavelength 600nm experiences maximum reflection above the layer of oil?
6. A thin layer of oil floats on a puddle of water. White light falls normally onto a thin oil film with refractive index 1,5. At what minimum thickness of the film will the reflected rays be coloured green ($\lambda_g = 550\text{nm}$)?