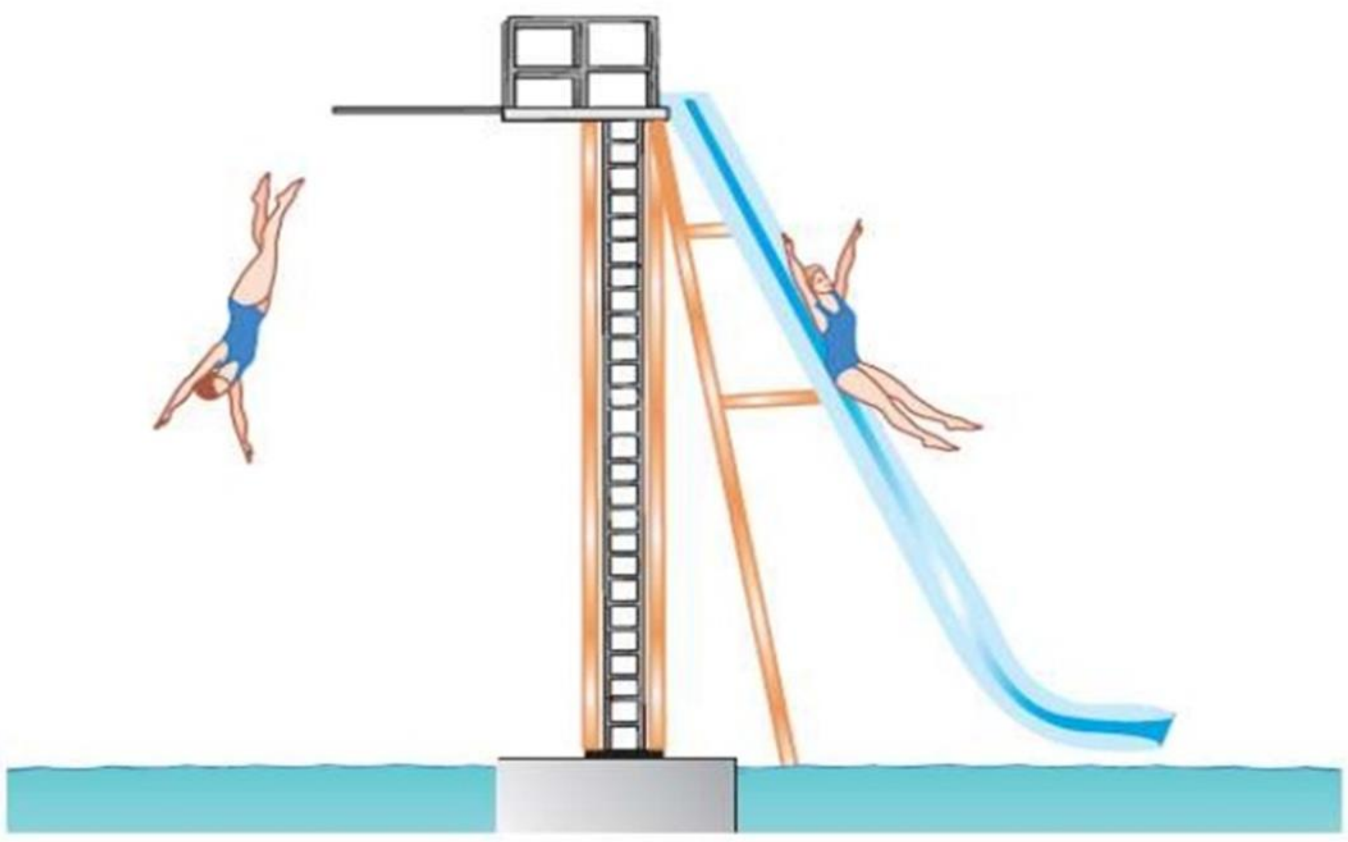
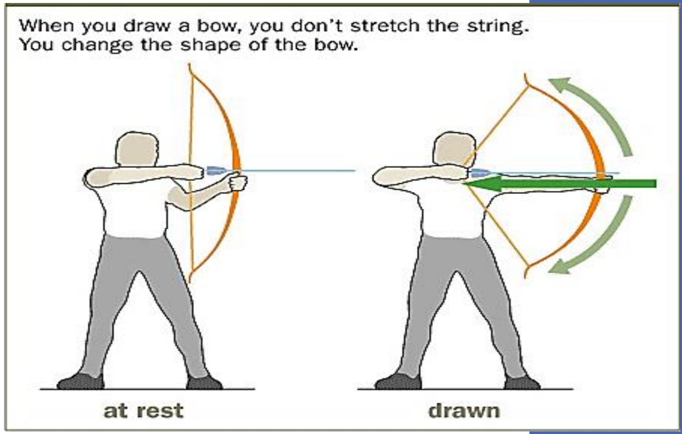


POTENTIAL ENERGY

POTENTIAL ENERGY

The term potential energy was introduced by the 19th century Scottish engineer and physicist William Rankine although it has links to Greek philosopher Aristotle's concept of potentiality.

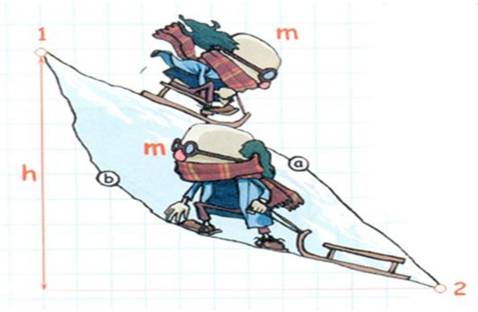
An object can store energy as the result of its position. For example a drawn bow is able to store energy as the result of its position. When assuming its usual position (at rest), there is no energy stored in the bow. Yet when its position is altered from its usual equilibrium position, the bow is able to store energy by virtue of its position (picture 1). This stored energy of position is referred to as potential energy. **Potential energy** is the stored energy of position possessed by an object.



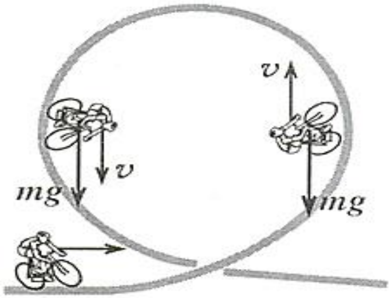
picture 1

CONSERVATIVE FORCES

Conservative forces are any force where the work done by that force on an object only depends on the initial and final positions of the object. In other words, the work done by a conservative force on on object does not depend on the path taken by the object.(picture 2)



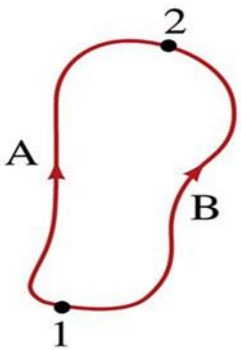
picture 2



picture 3

Equivalently, if an object travels in a circle (picture 3), the resultant work done by a conservative force is zero (this means that in one part of the circle gravity performs positive work, and in the second part of the circle gravity performs negative work)

Because the work done by gravity doesn't depend on the path taken, we call gravity a conservative force. other examples of conservative forces are: force in elastic spring, electrostatic force between two electric charges, magnetic force between two magnetic poles.



picture 4

When the body moves from position 1 to position 2 (picture 4), the work of conservative forces is equal to the difference of potential energy of the body in the initial and final position.

$$W = E_{P1} - E_{P2}$$

When an object moves from one location to another, the force changes the potential energy of the object by an amount that does not depend on the path taken.

TYPES OF POTENTIAL ENERGY

- **GRAVITATIONAL POTENTIAL ENERGY**
- **POTENTIAL ENERGY ON A LARGE DISTANCE FROM THE EARTH**
- **SPRING POTENTIAL ENERGY**

Example: An Earth satellite is in a circular orbit at an altitude of 500 km. Explain why the work done by the gravitational force acting on the satellite is zero?

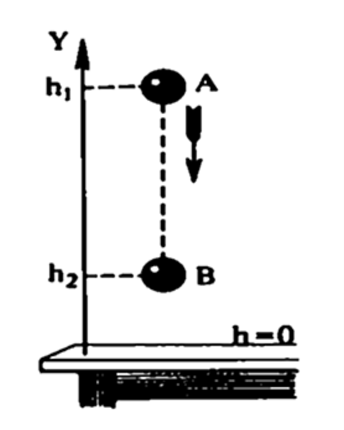
GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy is the energy stored in an object as the result of its vertical position or height. The energy is stored as the result of the gravitational attraction of the Earth for the object.

Here are some example of gravitation potential energy:

- A waterfall
- A suspension bridge
- Diver stands at the end of a diving board
- The car at the top of roller coaster (picture 5)

picture 5



picture 6

When a body moves vertically downwards (picture 6) the direction of the force of gravity coincides with that of the displacement. When the body moves from the height h_1 above a certain level to the height h_2 above the same level (picture), the magnitude of its displacement is $h_1 - h_2$. Since the direction of the displacement and force coincide, the work done by the force of gravity is positive:

$$\mathbf{W} = \boldsymbol{mg(h_1 - h_2)} \Rightarrow \mathbf{W} = \boldsymbol{mgh_1 - mgh_2} \ldots (\mathbf{1})$$

According to the definition of potential energy, we have: $\mathbf{W} = \boldsymbol{E_{P1} - E_{P2}} \ldots (\mathbf{2})$

From the given formulas (1) and (2) it can be concluded that:

$$\boldsymbol{E_{P1} = mgh_1} \qquad \boldsymbol{E_{P2} = mgh_2}$$

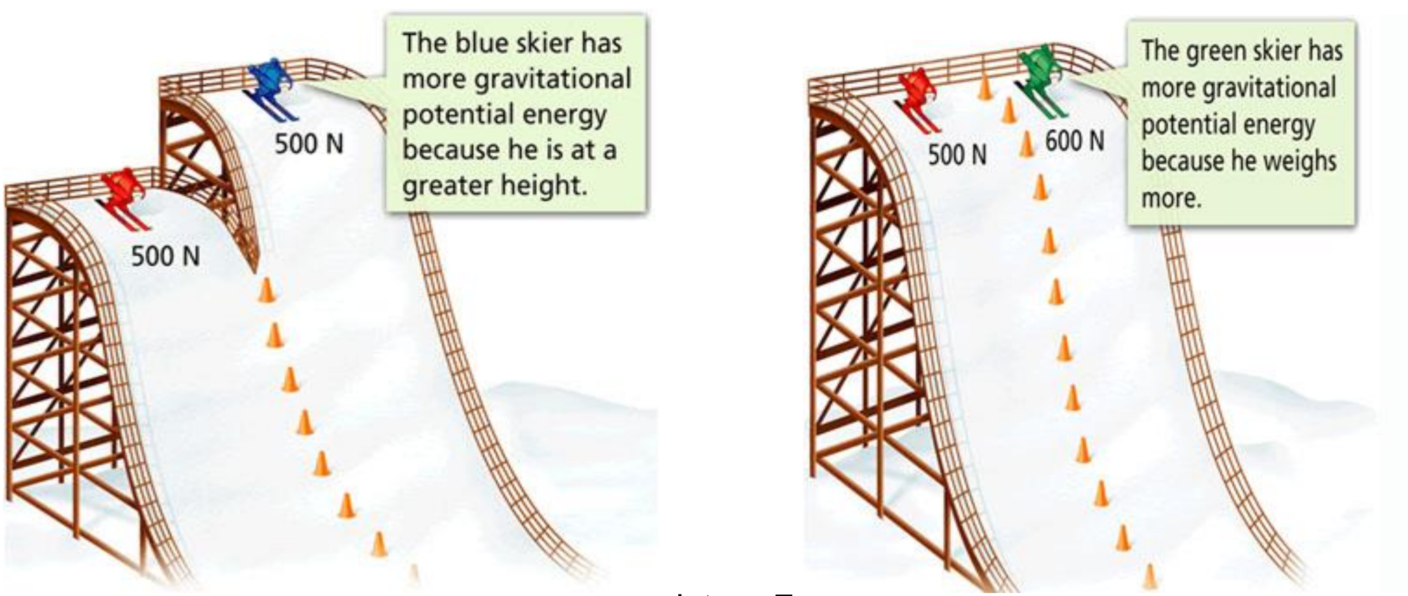
We can generalize this conclusion

Potential energy of a body of mass m at a height h above the ground level is given by:

$$E_P = mgh$$

This suggest that an object's gravitational potential energy depends on two factors:

- the object's height h above ground level –the greater its height, the greater its gravitational potential energy
- the object's mass m –the heavier the object, the greater its gravitational potential energy (picture 7)



picture 7

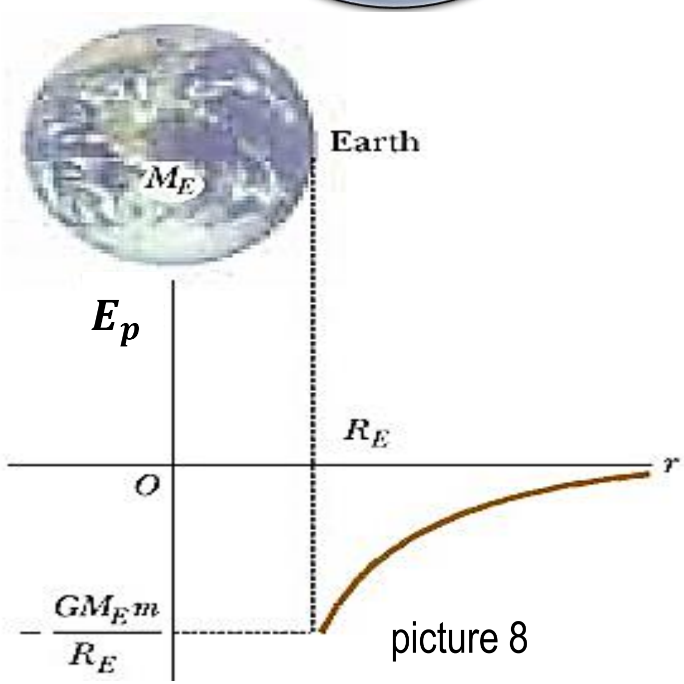
POTENTIAL ENERGY AT A LARGE DISTANCE FROM THE EARTH

The formula given for potential energy $E_P = mgh$ is only valid when the height h is much smaller than the radius of the earth, only in this case the force of gravity is constant so that its work can be calculated by the formula: $A = mg(h_1 - h_2)$

In general, the gravitational force is not constant, it decreases with the square of the distance from the center of the Earth.

$$\boldsymbol{F_g = G\frac{Mm}{r^2}}$$

where G-is the gravitation constant, M-the mass of the earth, m-the mass of the body, r- distance of the body from the earth

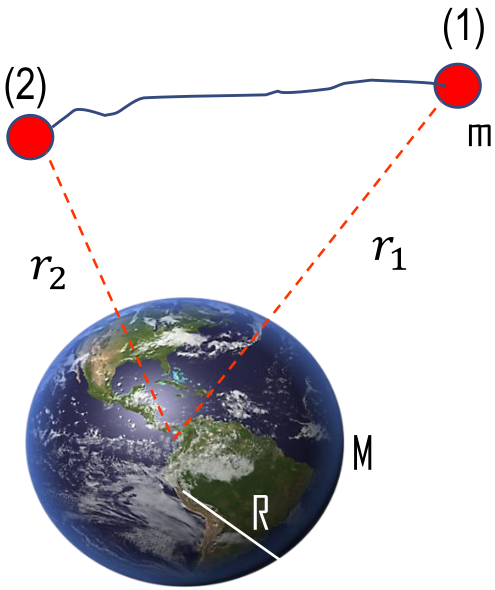


picture 8

Gravitational potential energy of a body of mass m at a distance r from the center of the earth is determined by the formula:

$$E_P = -G\frac{Mm}{r}$$

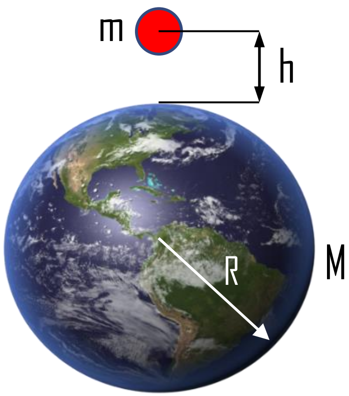
This formula applies to the Earth-object system where the two masses are separated by a distance r, provided that $r \geq R_E$ (picture 8). The result is not valid for bodies inside the Earth, where $r < R_E$. The choice of a reference level for the potential energy is point where the force is zero. (r=∞, $E_p = 0$).



picture 9

When the body moves from position 1 to position 2 (picture 9), the work of gravitational force is equal to the difference of potential energy of the body in the initial and final position.

$$W = E_{P1} - E_{P2} \Rightarrow W = -G\frac{Mm}{r_1} + G\frac{Mm}{r_2}$$



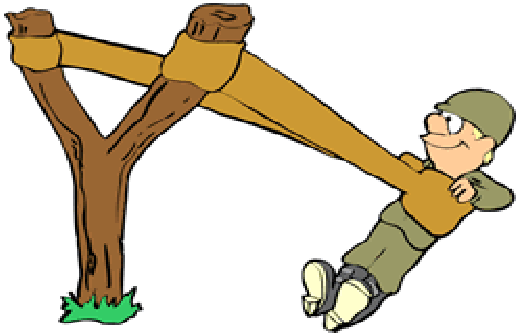
picture 10

If the body mass m is located at a height h above the earth's surface ,and R is the radius of the Earth, then the formula of the gravitational potential energy is:

$$E_P = -G\frac{Mm}{(R+h)}$$

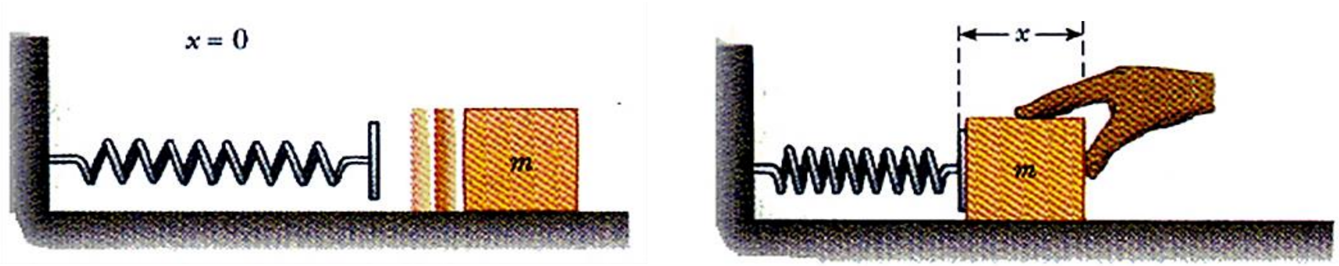
This negative potential energy is indicative of a "bound state"; once a mass is near a large body, it is trapped until something can provide enough energy to allow it to escape.

ELASTIC POTENTIAL ENERGY



picture 11

Elastic potential energy is the energy stored in elastic materials as the result of their stretching or compressing (picture 11).
Work done by an applied force in stretching or compressing a spring can be recovered by removing the applied force, so like gravity, the spring force is conservative.
Elastic potential energy can be stored in rubber bands, bungee chords, trampolines, springs, an arrow drawn into a bow, etc.



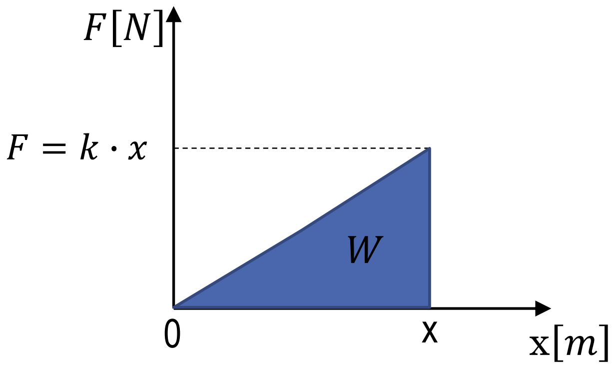
(a) (b)

picture 12

- Picture (a) shows a spring in its equilibrium position (x=0), where the spring is neither compressed nor stretched.

- Pushing a block of mass m on a frictionless surface against the spring as in picture (b) compresses it a distance *x*. Although *x* appears to be merely a coordinate, for springs it also represents a displacement from the equilibrium position.

Elastic potential energy is equal to the work done to compress the spring, it depends upon the spring constant k as well as the distance stretched (x). According to Hooke's law the force required to stretch the spring will be directly proportional to the amount the stretch (the more compression there is, the more force that is required to compress it further). Since the force has the form: $\boldsymbol{F = kx}$



picture 13

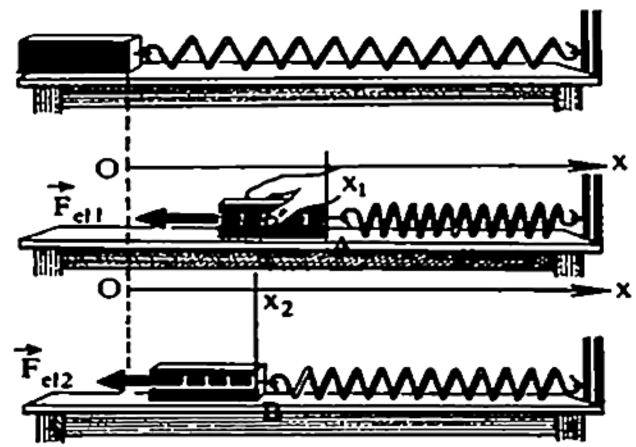
The picture 13 shows variation of elastic force as function of displacement x. Then the work done to compress the spring for distance x is equal to area under graph:

$$E_P = W = \frac{kx^2}{2}$$

So the elastic potential energy stored in the spring is equal to the work. The amount of elastic potential energy E_P stored in such a device (spring) is related to the amount of stretch of the device (spring) - the more stretch, the more stored energy. The elastic potential energy stored in the spring is zero when the spring is in the equilibrium position (x=0).

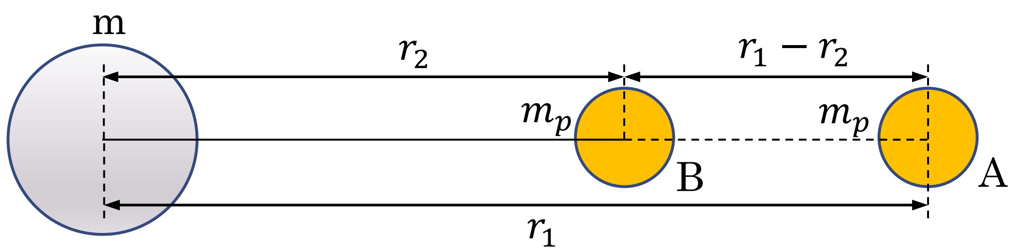
In general, when the spring is stretched from position x_1 to position x_2 (picture 14) the work done by the spring is equal to the difference of potential energy of the body in the initial (x_1) and final position (x_2).

$$\mathrm{W} = E_{P1} - E_{P2} \Rightarrow \mathrm{W} = \frac{kx_1^2}{2} - \frac{kx_2^2}{2}$$



picture 14

GRAVITATIONAL POTENTIAL



picture 15

Imagine we have a fixed mass *m* (gravitating object) and some test body with mass m_p that is moved from position *A* to position *B* by the gravitational force of m (picture 15).

If a test body with mass m_p has gravitational potential energy E_{pA} at some location A (relative to a choice of zero point of gravitational potential energy), then the gravitational potential at that location A is:

$$V_A = \frac{E_{pA}}{m_p}$$

$$V_A = \frac{E_{pA}}{m_p} \quad E_{PA} = -G\frac{mm_p}{r_1} \quad V_A = -G\frac{m}{r_1}$$

Where G-is the gravitation constant, and *m* is the mass of the gravitating object

We conclude that:
the gravitational potential is directly proportional to the mass of the gravitating object m and it is inversely proportional to the distance of the test body from the gravitating object (this quantity is independent of the test body; so it can be regarded as properties of the "background" gravitational field alone).

PROBLEMS

1. A 500 g ball is dropped from the edge of a cliff 80 m from a ground. Find kinetic and potential energy of a ball after 3 seconds of free fall ?

2. A 500 g ball is thrown upward from the ground with initial velocity $10\frac{m}{s}$? Find the potential energy of the ball at the maximum height and half value of maximum height.

3. A 100 g stone is launched horizontally from the height of 20 m above the ground. Find the potential energy in initial moment and after time of 1 s.

4. A spring has an elastic potential energy of 4 J when compressed 10 cm.
   a)What is its spring constant ?
   b) What extra work required to compressed it an additional 10 cm ?

5. Find the potential energy of satellite of mass m=1 t in circular orbit around the Earth at the distance r=2R from the centre of the Earth.(The radius of the Earth is $R = 6{,}4 \cdot 10^6 m$). Free-fall acceleration at the surface of the Earth is $g_0 = 9{,}81\frac{m}{s^2}$

7.What is the work done by gravitational force in moving an object of mass m=5kg from the surface of the Earth to infinite height/infinity?

6. Show that the total energy of a satellite of mass m in circular orbit around the Earth (mass M) at a distance r=5R from the Earth's centre (R-radius of Earth) is given by: $E = -G\frac{mM}{10R}$