LAW OF CONVERSATION OF ENERGY

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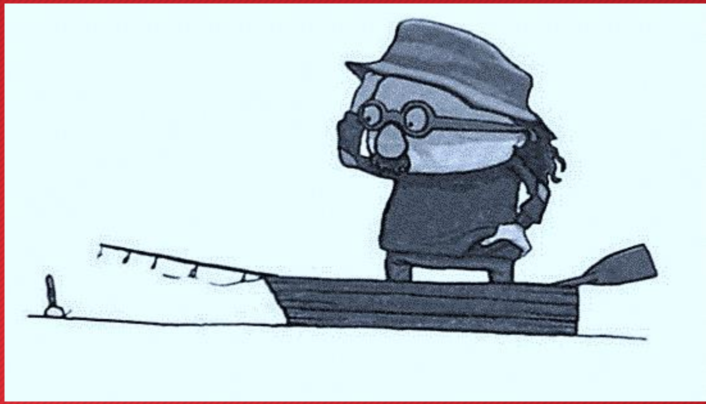
The law of conservation of energy is true for isolated (closed) systems of the body.
An isolated system is a collection of two or more bodies that interact only with each other. An isolated system is sistem that does not interact with its surroundings.



Isolated system (man-boat)

LAW OF CONVERSATION OF ENERGY (ISOLATED SYSTEMS)

Let us consider how the energy of bodies interacting only with one another changes. It should be recalled that such bodies form a closed system. Interacting bodies may have kinetic and potential energy simultaneously.

For example an artificial satellite of the Earth has a kinetic energy as a moving body. Besides the satellite-Earth has a potential energy since the satellite and the Earth interact through the force of universal gravitation (picture).

If bodies forming a closed system interact they move relative to one another. In this motion, both their velocities and coordinates may change. Consequently, the kinetic energy of the bodies, as well as their potential energy may change.

Let us denote by E_{P1}, the potential energy of interacting bodies at a certain instant of time and by E_{K1} their total kinetic energy at the same instant. The potential and kinetic energies of the same bodies at some other instant of time be respectively E_{P2} and E_{K2}

When the bodies interact through an elastic force or a force of gravity, then the work W done by these forces is equal to the change in the potential energy of the bodies with the opposite sign:

$$W= -(E_{P2} - E_{P1})......(1)$$

On the other hand, according to the kinetic energy theorem, the work done by the same force is equal to the change in the kinetic energy.

$$W= E_{K2} - E_{K1}.....(2)$$

Comparing (1) and (2) we see that the change in the kinetic energy is equal in magnitude to the change in the potential energy, but they have opposite signs:

$$E_{K2} - E_{K1} = -(E_{P2} - E_{P1}).....(3)$$

If the potential energy of bodies increases, their kinetic energy decreases by the same value, and vice versa. Hence it is clear that one type of energy is converted into the other.

Obviously formula (3) can be written in a different form:

$$E_{K2} + E_{P2} = E_{K1} + E_{P1}$$

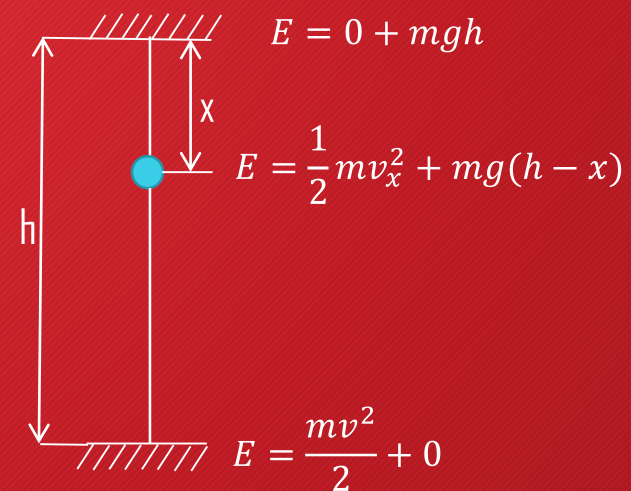
Thus the sum of the kinetic and potential energies of bodies constituting a closed system and interacting through forces of universal gravitation or elastic forces remains constant. This is essence of the law of conservation of energy.

$$E_K + E_P = const \Rightarrow E = const$$

The sum of the kinetic and potential energies of a system of bodies is usually called the total mechanical energy. The total mechanical energy of a closed system of bodies interacting through the forces of gravity or elastic forces remains unchanged.

Example: free falling body from a height h

- $E = E_P = mgh$
- $E = \frac{1}{2}mv_x^2 + mg(h-x) = \frac{m}{2}2gh + mg(h-x) = mgh$
- $E = E_K = \frac{mv^2}{2} = \frac{m}{2}2gh = mgh$

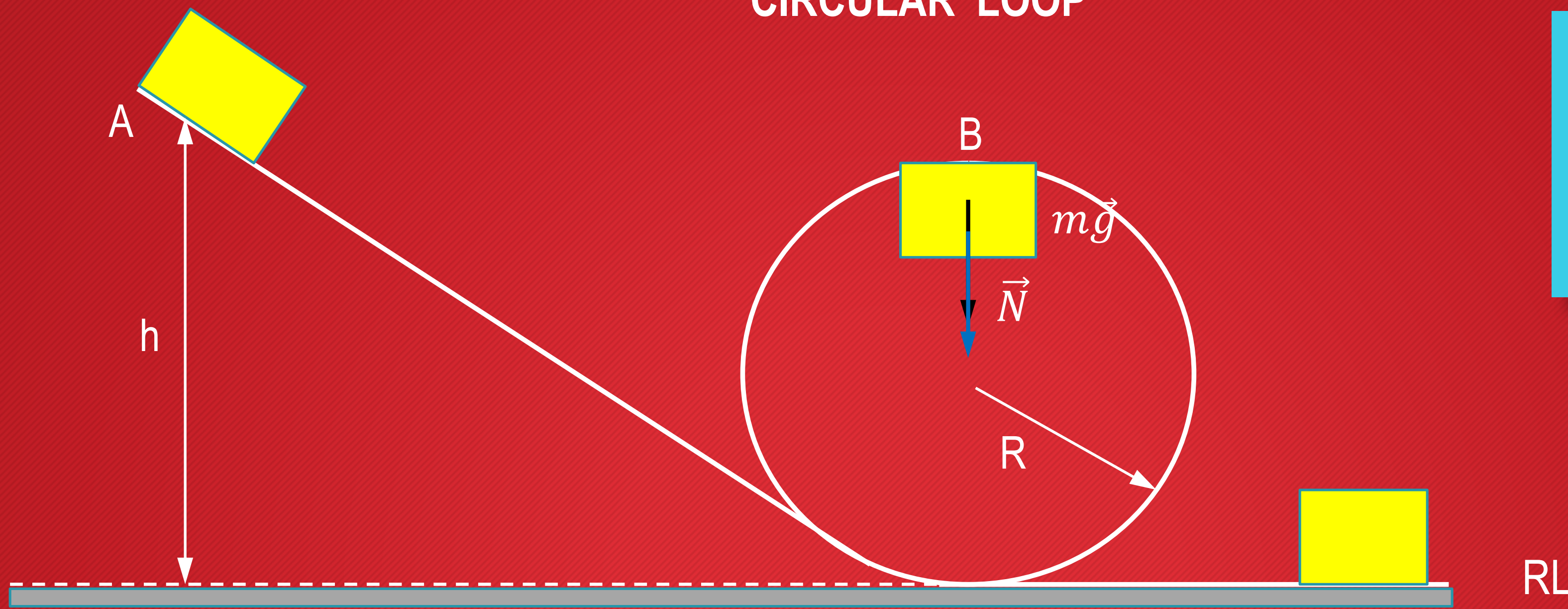


$E_{K1} + E_{P1} = E_{K2} + E_{P2}$ $\qquad$ $E_K + E_P = const$

Changing the mechanical energy in this case is equal to zero:

$$\Delta(E_K + E_P) = 0$$

CIRCULAR LOOP



Example: What is the minimum height that a mass can be released from rest and still make it around the circular loop without falling off? Neglect friction.

First, we need to know the minimum speed at the top of the circular loop for the mass to remain on the track.. At the top of the circular loop (point B), the two forces are N and mg, both acting down.
$ma_{cp} = \text{mg} + \text{N}$ ➡ N=mv^2/R-mg ➡ Minimum speed is giveb by: N=0 so v^2=Rg
Multiply by 1/2m we get: 1/2mv^2=1/2Rg. So the minimum kinetic energy at the top is ½ Rg.
Using the law of conservation of energy for points A and B we calculate:
E_A=E_B mgh=1/2mv^2+mg2R (The referent level for potential energy is the bottom of the circular loop).
Finally h=R+R/2=5/2R
A mass released from lower than h = 5R/2 will fall off the loop. A mass released from higher up will have a non-zero normal force at the top, and will loop the loop.

LAW OF CONVERSATION OF ENERGY (NON-ISOLATED SYSTEMS)

Up to now, we have only considered isolated systems, the mechanical energy is constant. Enegy is not lost or gained by the system, as long as only conservation forces are involved. Let us now generalize our approach to deal with non-isolated systems, if non conservative forces are involved energy must be added to the system or removed from it.

Internal forces - all the forces acting on the system originate within the system itself

External force – all the forces originate outside the system

Consider an arbitrary system of the body which passes from a position 1 to position 2. Let us denote by W_K work of all internal conservative forces
and by W_E the work of all internal non−conservative and all external forces

Total work in the system of all the forces on the body is: W$= W_K + W_E$

Let us denote by E_{K1} the total kinetic energy of the body in the position 1
and by E_{K2} the total kinetic energy of the body in the position 2

According to the theorem work-energy :

W$= E_{K2} - E_{K1}$ → $W_K + W_E = E_{K2} - E_{K1}$ → $W_E = E_{K2} - E_{K1} - W_K$

$W_K = E_{P1} - E_{P2}$

$W_E = E_{K2} - E_{K1} - E_{P1} + E_{P2}$ → $W_E = (E_{K2} + E_{P2}) - (E_{K1} + E_{P1})$ → $W_E = E_2 - E_1$

We find that the work done on a system by all external forces and non-conservative forces is equal to the change in mechanical energy of that system.

$$W_E = E_2 - E_1$$

- If the total work of external and non-conservative force is positive, then the mechanical energy of the system increases
- If the total work of external and non-conservative force is negative, then the mechanical energy of the system decreases, the system consumes energy to overcome a force that opposes motion.

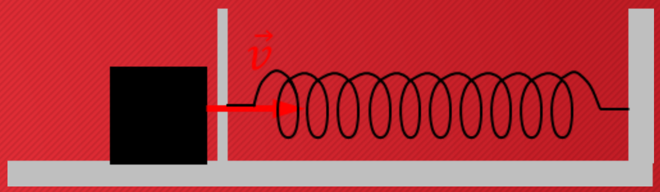
Trampoline jumping involves an interplay among kinetic energy, gravitational potential energy, and elastic potential energy. Due to air resistance an frictional forces within the trampoline, mechanical energy is not conserved. That's why the bouncing eventually stops unless the jumper does work with his or her legs to compensate for the lost energy.

PROBLEMS

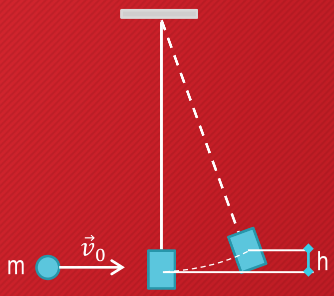
1. A 50 g stone is thrown downward with initial velocity $10\frac{m}{s}$ in water from a height of 10 m. Find kinetic and potential energy of stone 3 m above the surface of water.

2. A ball is thrown downward from a height of 2 m with an initial velocity v_0. When the ball strikes the floor, it jumps up to a height 4 m. Find initial velocity of the ball ?

3. A block with mass of 800 g hits horizontal spring (with spring constant 10kN / m) with initial speed 4m/s. The other end of the spring is fixed to the wall (picture).Find the compression of the spring?

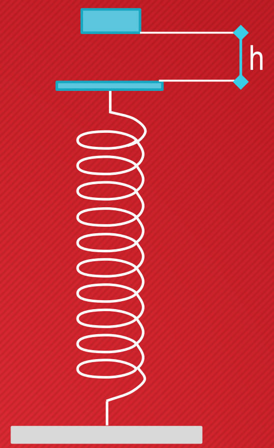


4. A bullet of mass m=10 g flies horizontally with velocity $400\frac{m}{s}$ hits a sandbag with mass M =1990 g and stucks in it. Find:
   a) velocity of sandbag (with bullet) after impact
   b) at what height does the sandbag rise ?



5. A body with a mass of 4kg moves with a velocity of 9 *m/s* and strikes an immobile body with a mass of 8kg .Find the reduction in the kinetic energy after impact (in percent)?

6. A body of mass m fall from a height h onto the pan of a spring balance. The mass of the pan and the spring are negligible. Having a stuck to the pan, the spring is compressed for the value h. Find the spring constant k of the spring.



7. A man standing on a cart at rest throws a 600 g stone in a horizontal direction with initial velocity 15 m/s. The cart with the man rolls 8cm backwards.Mass of man with card is 50kg. Find the coefficient of kinetic friction between the cart and the floor?

8. A 1 kg stone is dropped vertically from a height 5m. The stone strikes the floor with velocity $9\frac{m}{s}$. Find work done by resistance force during the motion of the stone? Find the average value of resistance force?

9. A 1kg stone is thrown vertically downward with initial speed of 14m/s from a height of 240m and penetrates the sand to a depth of 0,2m. Find the resistence force that act on a stone in a sand?

10. A 30 kg sledge slides down a slope of height 5m and travels a distance of 20m. If sledge's initial velocity (at the top od the slope) is equals to 0 and final velocity (at the end of the slope) is 4,5m/s, what is the friction force acting on the sledge as it moves down the slope. The sledge reaches the bottom of the slope continuing on the horizontal ground and comes to a stop after 4,5m. What is the friction force acting on the sledge on the horizontal ground?