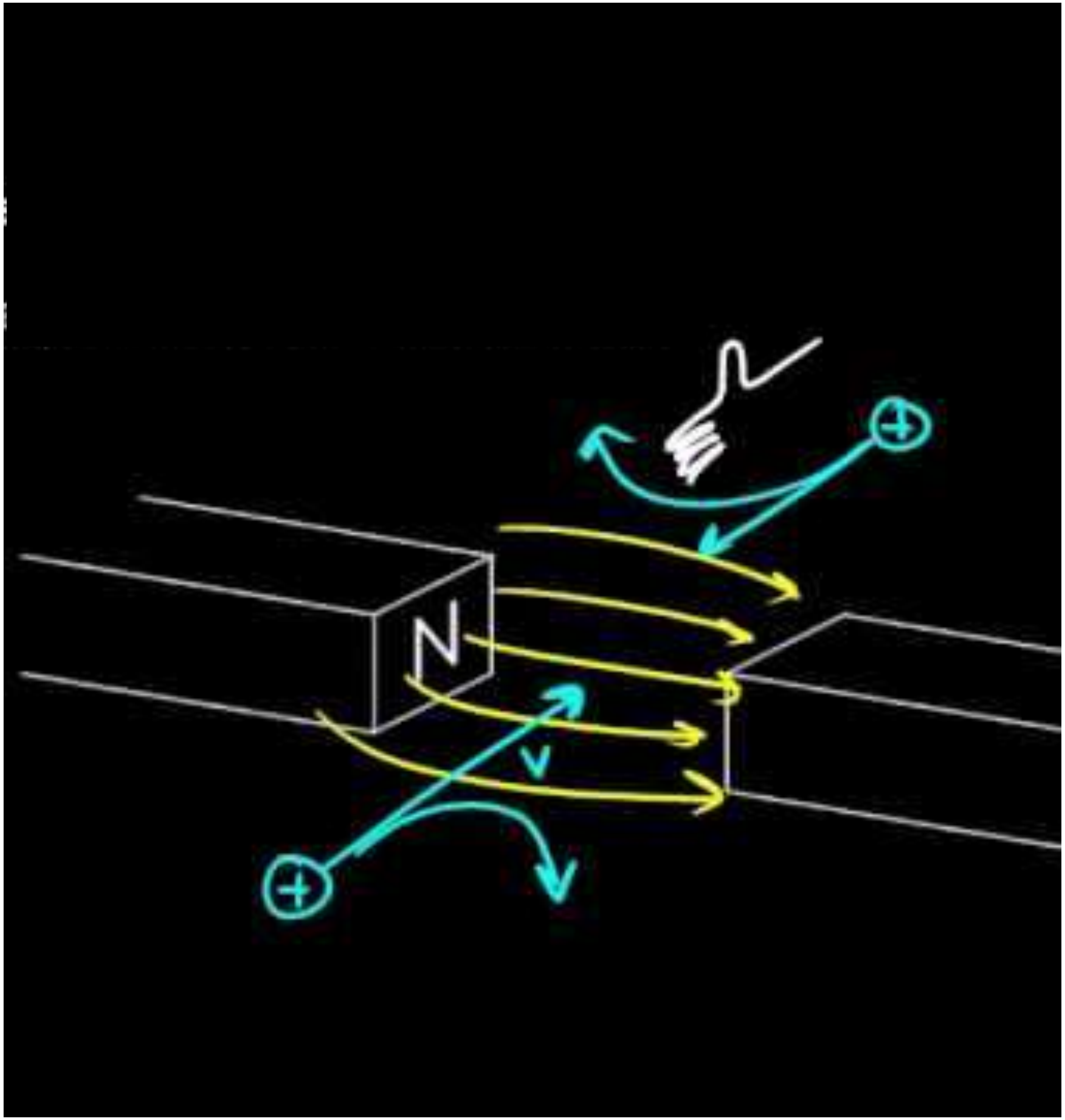

LORENTZ MAGNETIC FORCE

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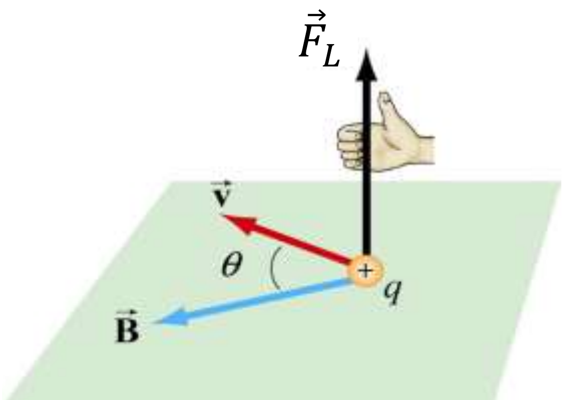
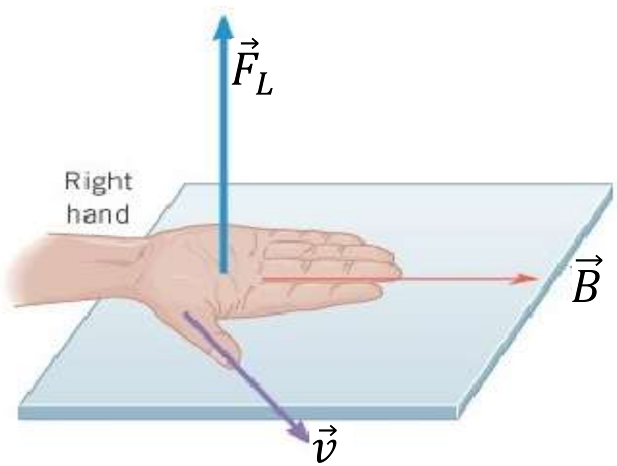
The force which is exerted by a magnetic field on a moving charge particle is called Lorentz magnetic force and it is given by formula:

$$\vec{F}_L = q\vec{v}x\vec{B}$$

Where $\vec{F}_L$ is the Lorentz magnetic force vector, q is the charge of the moving particle, $\vec{v}$ is the velocity vector of the moving particle, and $\vec{B}$ is the magnetic field vector.



picture 1



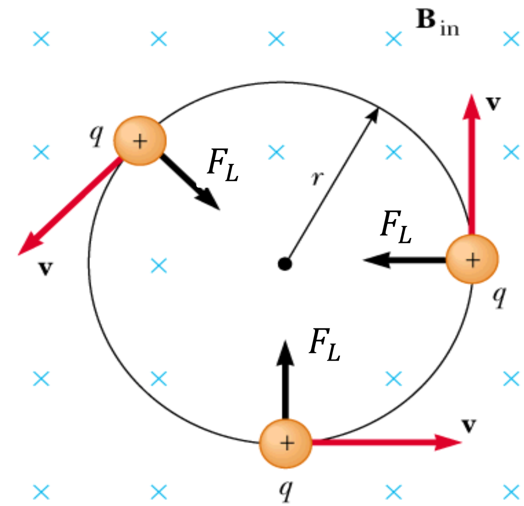
picture 2

The implications of this expression include:

1. The Lorentz magnetic force is perpendicular to both the velocity v of the charge q and the magnetic field B.
2. The direction of the Lorentz magnetic force can be found using the right-hand-slap rule (picture 2). This rule describes the direction of the Lorentz magnetic force as the direction of a 'slap' of an open hand. As with the right-hand-grip rule, the fingers point in the direction of the magnetic field. The thumb points in the direction that **positive** charge is moving. If the moving charge is negative (for example, electrons) then you need to reverse the direction of your thumb because the force will be in the opposite direction. Alternatively, you can use your left hand for moving **negative** charge.
3. The magnitude of the Lorentz magnetic force is $\boldsymbol{F = qvBsin\theta}$ where θ is the angle < 180 degrees between the velocity and the magnetic field (picture 1). This implies that the Lorentz magnetic force on a stationary charge or a charge moving parallel to the magnetic field is zero.

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Consider the case of a positively charged particle moving in a uniform magnetic field so that the direction of the particle's velocity is perpendicular to the uniform magnetic field, as in picture 3. The Lorentz magnetic force causes the particle to alter its direction of travel and to follow a curved path. Application of the righthand rule at any point shows that the magnetic force is always directed toward the center of the circular path; therefore, the magnetic force causes a centripetal acceleration, which changes only the direction of $\vec{v}$ and not its magnitude. Because $\vec{F}_L$ produces the centripetal acceleration, we can equate its magnitude, $F_L = qvB$ in this case, to the mass of the particle multiplied by the centripetal acceleration v^2/r. From Newton's second law, we find that:



picture 3

$$ma_c = F_L \Rightarrow m\frac{v^2}{r} = qvB$$

Where r is the radius of the circle the charged particle will move on. Therefore:

$$r = \frac{mv}{qB}$$

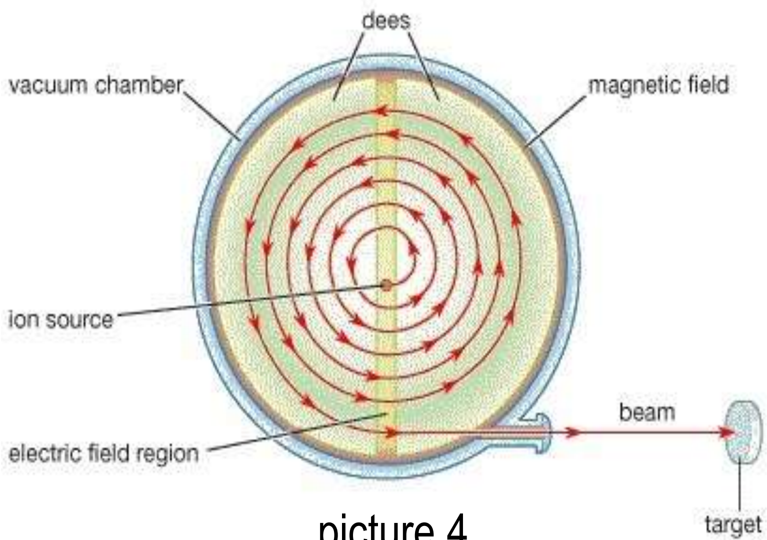
Very massive or very fast charges will move on large circles, large charges and large magnetic field will result in small circles. Time to make one full revolution in a magnetic field is formed from:

$$T = \frac{2r\pi}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

► Particle accelerator

Any device that accelerates charged particles to very high speeds using electric and/or magnetic fields is particle accelerator.

A **cyclotron** is a type of particle accelerator invented by Ernest O. Lawrence in 1929-1930 at the University of California, Berkeley and it is still used as the first stage of some large multi-stage particle accelerators.

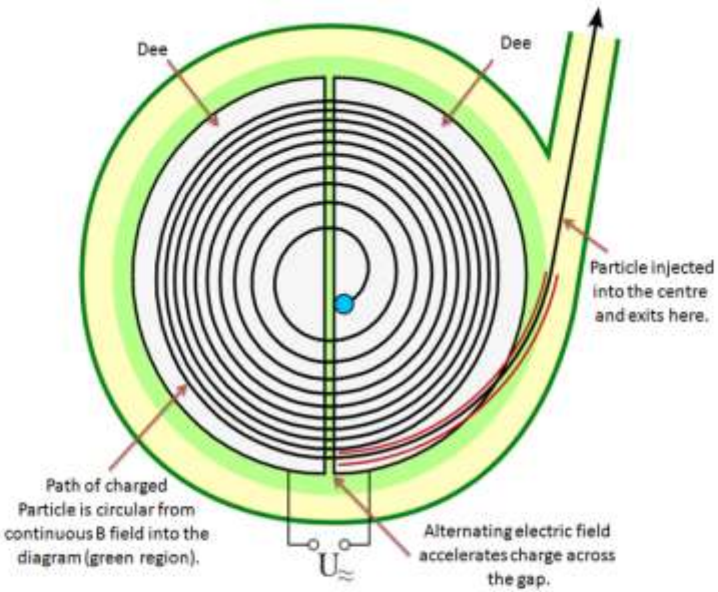


picture 4

A cyclotron accelerates a charged particle beam using a high frequency alternating voltage which is applied between two hollow "D"-shaped sheet metal electrodes called "dees" inside a vacuum chamber The dees are placed face to face with a narrow gap between them, creating a cylindrical space within them for the particles to move. The particles are injected into the center of this space. The dees are located between the poles of a large electromagnet which applies a static magnetic field B perpendicular to the electrode plane.

Let a particle of charge *q* and mass *m* enter a region of magnetic field $\vec{B}$ with a velocity $\vec{v}$ normal to the field $\vec{B}$. The magnetic field causes the particles' path to bend in a circle due to the Lorentz force perpendicular to their direction of motion, the necessary centripetal force begin provided by the magnetic field. Therefore, the Lorentz magnetic force on charge *q* is equal to the centripetal force on charge *q*:

$$qvBsin90° = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$



picture 5

Therefore, each time the particles cross the gap from one dee electrode to the other, the electric field is in the correct direction to accelerate them (picture 5). The particles' increasing speed due to these pushes causes them to move in a larger radius circle with each rotation, so the particles move in a spiral path under the influence of a constant magnetic field outward from the center to the rim of the dees. Period of revolution of the charged particle is given by:

$$T = \frac{2r\pi}{v} \Rightarrow r = \frac{mv}{qB} \Rightarrow T = \frac{2m\pi}{qB}$$

When they reach the rim a small voltage on a metal plate deflects the beam so it exits the dees through a small gap between them, and hits a target located at the exit point at the rim of the chamber, or leaves the cyclotron through an evacuated beam tube to hit a remote target.

The particles reach their maximum energy at the periphery of the dees, where the radius of their path is $r = r_n$ the radius of the dees. The particles will attain maximum velocity v_n near the periphery of the dees:

$$qv_nB = \frac{mv_n^2}{r_n} \Rightarrow v_n = \frac{qBr_n}{m}$$

The maximum kinetic energy of the particles will be:

$$E_k = \frac{mv_n^2}{2} = \frac{q^2B^2r_n^2}{2}$$

Since the particles are accelerated by the voltage many times, the final energy of the particles is not dependent on the accelerating voltage but on the strength of the magnetic field and the diameter of the accelerating chamber the dees.

PROBLEMS

1. An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2 mT. If the speed of the electron is $1{,}5 \cdot 10^{7}$ m/s, determine
   (a)the radius of the circular path
   (b) the time it takes to complete one revolution.

2. Find the kinetic energy of a proton moving along a circle with a radius of 60 cm in a magnetic field having an induction of 5mT(in joules and electron-volts) ?

3. A proton and an electron move with the same velocity and penetrate into a homogeneous magnetic field. How many times is the radius of curvature of the path of the proton r_1 greater than that of the electron r_2?

4. A proton and an electron move with the same kinetic energies and penetrate into a homogeneous magnetic field. How many times is the radius of curvature of the path of the proton r_1 greater than that of the electron r_2?

5. A beam of electrons accelerated by a potential differenceof *V=300* V penetrate into a homogeneous magnetic field (with a magnetic induction of 0,92mT.) and moves in a circular path. Find:
   a) the velocity of an electron in a magnetic field
   b) the time it takes to complete one revolution.