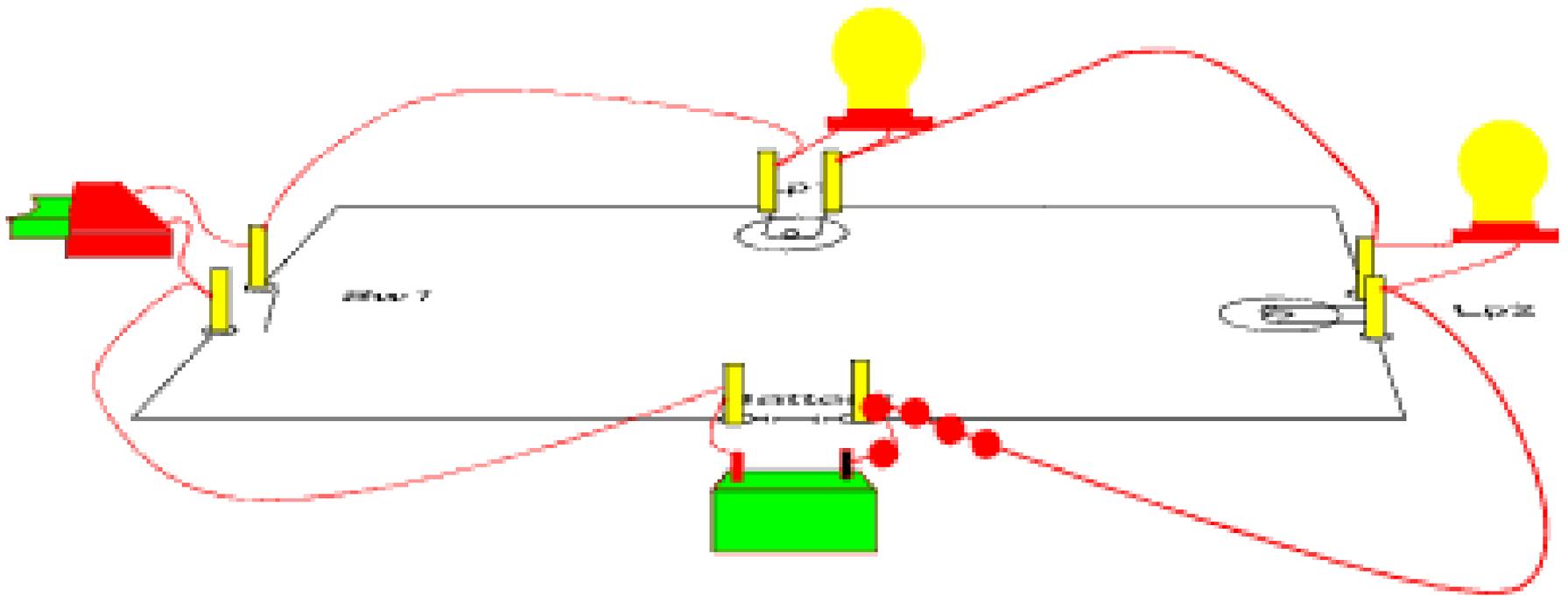


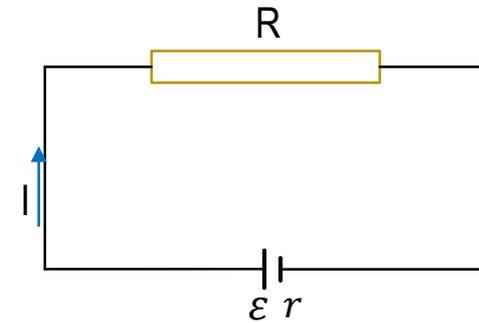
# OHM LAWS IN CLOSED ELECTRICAL CIRCUIT

## KIRCHHOFF'S LAWS



## OHM LAWS IN CLOSED ELECTRICAL CIRCUIT

Picture 1 shows the schematic for a simple circuit. A simple circuit has a single voltage source and a single resistor. The cables connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in  $R$ . The electric current through the simple circuit depends on the electromotive force of the source (battery)  $\varepsilon$  and the resistances (external resistance  $R$  and internal resistance of the cell  $r$ ) in the simple circuit.



picture 1

The total energy transferred by the source (battery) must equal the amount of energy transferred to the other components in the circuit (energy is always conserved).

- ◆ When charge flows through a source (battery) it is given electric energy by the source. This energy is equal to the work done by external forces.  $\Rightarrow W_s = \varepsilon q = \varepsilon I t$
- ◆ Resistors release heat:  $Q = I^2 R t + I^2 r t$
- Because energy is conserved in closed electric circuit then:

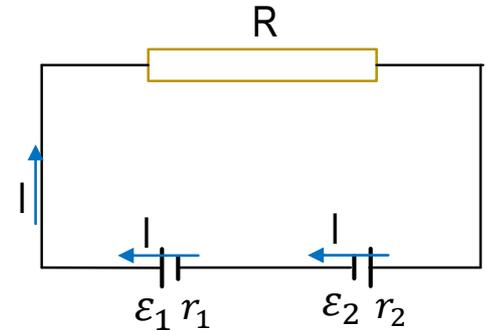
$$W_s = Q \Rightarrow \varepsilon I t = I^2 R t + I^2 r t \Rightarrow \varepsilon = I(R + r) \Rightarrow I = \frac{\varepsilon}{R + r}$$

This is known as Ohm's law for a closed circuit

**So the electric current intensity is equal to the electromotive force of the source divided by the total resistance of the circuit**

When the two electric sources in the circuit are connected in series (picture 2), and the current flows through the circuit in the direction indicated, then in the first source ( $\varepsilon_1$ ) the work done by external force is positive and in another source ( $\varepsilon_2$ ) the work done by external force is negative.

If we denote the electromotive force of the first source by ( $\varepsilon_1$ ), the electromotive force of the second source by ( $\varepsilon_2$ ), the total current intensity in the circuit by ( $I$ ), the external resistance by ( $R$ ), the internal resistance of the first source by ( $r_1$ ) and the internal resistance of the second source by ( $r_2$ ). Because energy is conserved in closed electric circuit then:



slika 2

$$W_{s1} - |W_{s2}| = I^2 R t + I^2 r_1 t + I^2 r_2 t \quad \Rightarrow \quad I = \frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2}$$

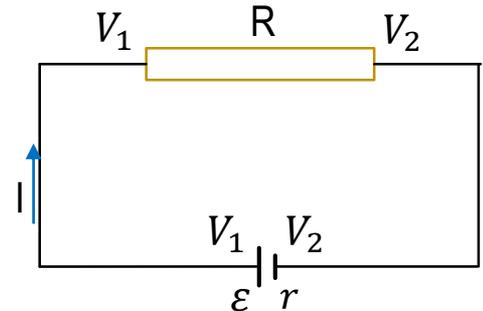
In this formula, the electromotive force of the source is taken with the sign '+' when the current flows from the negative pole to the positive pole and the electromotive force of the source is taken with the sign '-' when the current flows from the positive pole to the negative pole.

**So the electric current intensity is equal to the total electromotive force divided by the total resistance of the circuit.**

## ► Potential difference (voltage) between the poles of the source

In an closed circuit, the electromotive force must not be equal to the potential difference between the poles of the source so that the current flows (current only flows through a circuit when a voltage source is connected to it). In an open circuit, the electromotive force is equal to the potential difference between the poles of the source so that the current does not flow

Let's consider the simplest circuit (picture 3). The positive pole of the source and one end of the resistor  $R$  are connected directly with the wire and they have the same potential  $V_1$ , the negative pole of the source and the other end of the resistors are connected directly with the wire and they have the same potential  $V_2$  ((all points along the wire have the same potential))



picture 3

So, the potential difference ( $V$ ) between the positive and negative pole of the source is equal to the potential difference across the external resistance  $R$ :  $V_1 - V_2 = IR \dots (*)$

According to ohm's law:

$$\varepsilon = I(R + r) \Rightarrow \varepsilon = IR + Ir \dots (**)$$

**In first case, since the current flows from the negative pole to the positive pole inside the source then the potential difference between the positive and negative pole of the source becomes greater than the electromotive force.:**

$$V_1 - V_2 = \varepsilon - Ir$$

From (\*) and (\*\*) we get:  $\Rightarrow V_1 - V_2 = \varepsilon - Ir$

So the voltage between the poles of the source is less than the electromotive force.

**In second case, since the current flows from the positive pole to the negative pole inside the source, then the potential difference between the positive and negative pole of the source becomes less than the electromotive force.:**

$$V_1 - V_2 = \varepsilon + Ir$$

# KIRCHHOFF'S LAWS (KIRCHHOFF'S RULES)

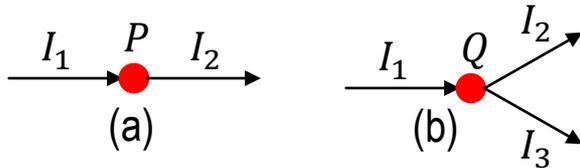
**Kirchhoff's laws** are two equalities that deal with the current and voltage in the electrical circuits. Kirchhoff's laws were first described in 1845 by German physicist Gustav Kirchhoff.

## ► Kirchhoff's first law (Kirchhoff's current law)

We should be familiar the idea that current may divide up where a circuit splits into two or more separate branches. The total amount of current remains the same after it splits. This is the bases of Kirchhoff's first law, which state that:



**The sum of the currents entering any point in a circuit is equal to the sum of the current leaving that same point**



picture 1

This is illustrated in picture 1-a. IN the first part, the current into point P must equal the current out so:  $I_1 = I_2$

In the picture1-b, we have one current coming into point Q and two current leaving. The current divides at Q, Kirchhoff's first law gives:

$$I_1 = I_2 + I_3$$

Kirchhoff's first law is an expression of the conservation of charge. The idea is that the total amount of charge entering a point must exit the point (if a billion electron enter a point in a circuit in a time interval of 1s, then one billion electron must exit the point in 1s).

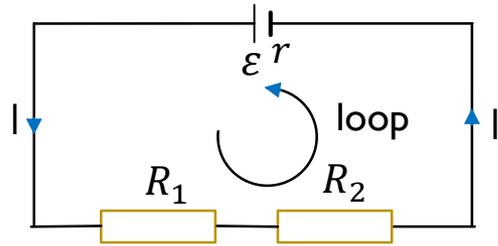
We can write Kirchhoff's first law as an eequation:

$$\sum I_{ent} = \sum I_{exit}$$

**The sum of all currents entering into a point in a circuit is equal to the sum of all current leaving that same point.**

## ► Kirchhoff's second law (Kirchhoff's voltage law)

This law deals with electromotive force (EMF) and voltage in a circuit.



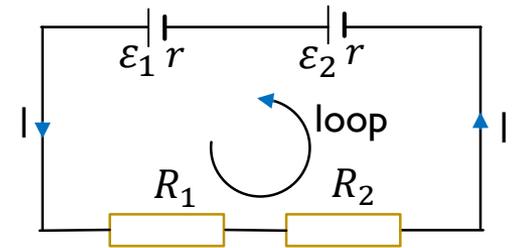
picture 2

We will start by considering a simple circuit which contains a cell and two resistors  $R_1$  and  $R_2$  (picture 2). Since this is a simple series circuit, the current  $I$  must be the same all way around. For this circuit we can write the following equation:  $\varepsilon = IR_1 + IR_2$

electromotive force of battery = sum of voltage across the resistors

We will considering a circuit which contains a two cell and two resistors  $R_1$  and  $R_2$  (picture 2). This is a series circuit so the current  $I$  is the same all way round the circuit. For this circuit we can write the following equation:

$$\varepsilon_1 + \varepsilon_2 = IR_1 + IR_2$$



picture 3

Kirchhoff's second law states that:

**The sum of the electromotive force around any loop in a circuit is equal to the sum of the voltage around the loop.**

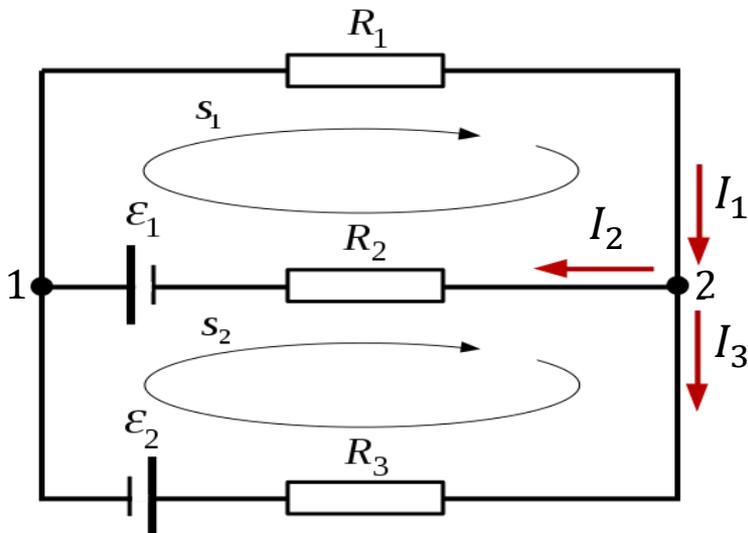

$$\sum \varepsilon = \sum V$$

Kirchhoff's first law is an expression of the conservation of energy.

## ► Applying Kirchhoff's laws

When applying Kirchhoff's rules, you must make two decisions at the beginning of the problem:

1. Assign symbols and directions to the currents in all branches of the circuit. Don't worry about guessing the direction of a current incorrectly; the resulting answer will be negative, but its magnitude will be correct.
2. When applying the loop rule, you must choose a direction for traversing the loop and be consistent in going either clockwise or counterclockwise. As you traverse the loop, record voltage drops and rises according to the following rules:
  - a) If a resistor is traversed in the direction of the loop, the voltage across the resistor is  $IR$
  - b) If a resistor is traversed in the direction opposite the loop, the voltage across the resistor is  $-IR$
  - c) If a source of emf is traversed in the direction of the emf (from - to + on the terminals), the voltage is  $+\varepsilon$ .
  - d) If a source of emf is traversed in the direction opposite the emf (from + to - on the terminals), the voltage is  $-\varepsilon$ .



picture 4

According to the Kirchhoff's first law at point 2 we have:

$$I_1 = I_2 + I_3$$

The Kirchhoff's second law applied to the closed circuit  $S_1$  gives:

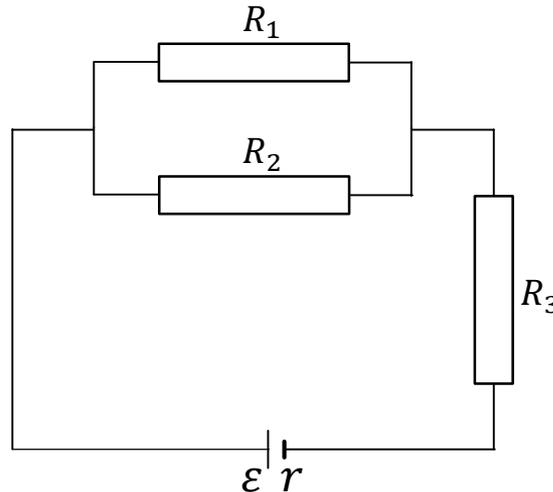
$$\varepsilon_1 = I_1 \cdot R_1 + I_2 \cdot R_2$$

The Kirchhoff's second law applied to the closed circuit  $S_2$  gives:

$$-\varepsilon_2 - \varepsilon_1 = I_3 \cdot R_3 - I_2 \cdot R_2$$

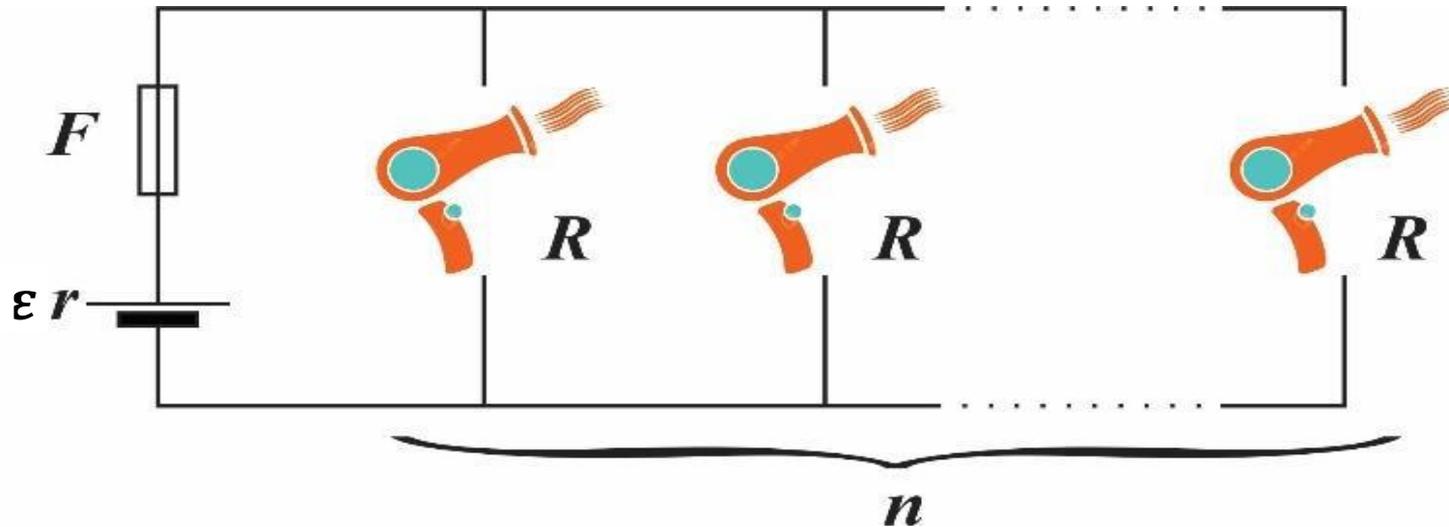
## PROBLEMS

1. An voltage source with an e.m.f. of 2 V has an internal resistance of  $0.5\Omega$ . Determine the potential difference at the ends of voltage source and external resistance of the circuit? Electric current of 0.25 A is flowing throught the external resistance R (draw a picture).
2. In the circuit (picture 1) voltage source with an e.m.f. equal to 4,5V has an internal resistance of  $0,25\Omega$ . The external resistances are  $R_1=5\Omega$ ,  $R_2=15\Omega$  and  $R_3=1\Omega$ . Find:
  - a) electric current flowing throught resistor  $R_3$
  - b) the potential difference at the ends of voltage source?



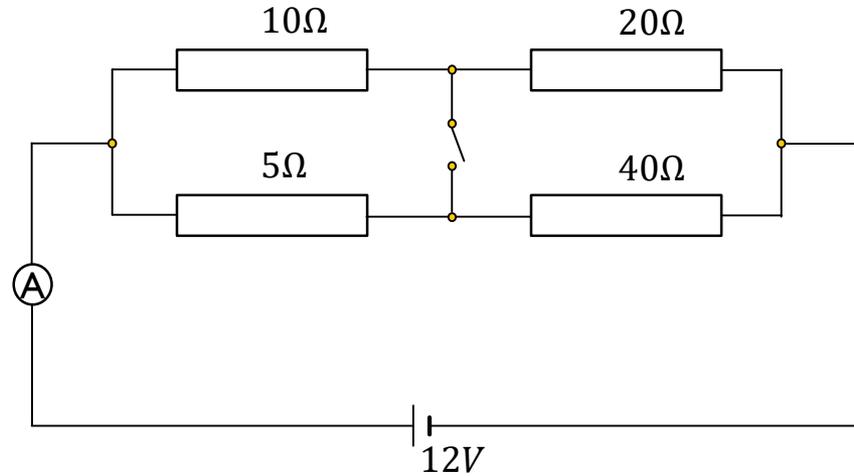
picture 1

3. Aleksandra and Ivan have decided to open a hairdresser's salon in which they plan on using a lot of hair dryers. Every hair dryer is acting as a resistor with resistance  $R=200\Omega$ . In order not to overload the circuitry, Ivan thought of connecting the hair driers to a voltage source  $\varepsilon=220V$  with an internal resistance of  $r=2,5\Omega$  electric fuse  $F$  with a negligible resistance. Electric fuse is a special element used in electric circuits in order to limit the máximum electric current allowed. When the current reaches the máximum value fuse blows, behaving like an open switch thus disabling the current flow. Maximum electric current allowed by the fuse Ivan and Aleksandra used in their hair salón is 16 A. Ivan asked Aleksandra to calculate the máximum number of hair dryers that can work simultaneously without causing the fuse to blow. What answer did Aleksandra give to Ivan? (picture 2)



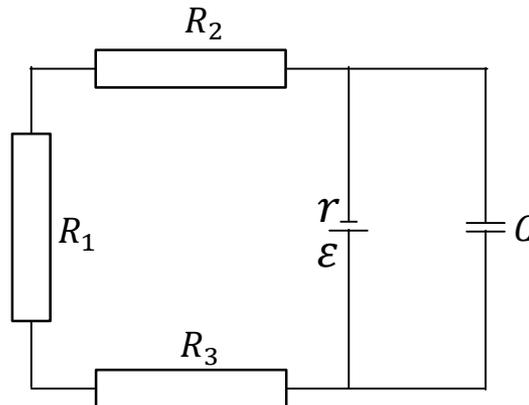
picture 2

4. Find the electric current passing through ammeter in the circuit (picture 3) when the switch is a) open b) close ?



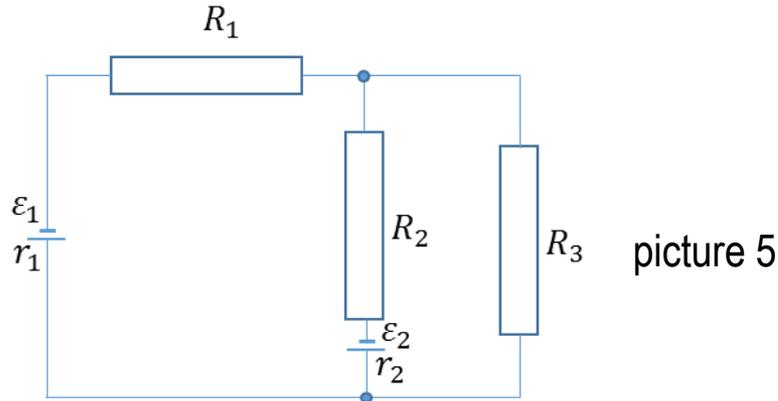
picture 3

5. In the circuit (picture 4) we have  $\varepsilon=3V$ ,  $r=0,5\Omega$ ,  $R_1=4\Omega$ ,  $R_2=2\Omega$ ,  $R_3=3,5\Omega$  and  $C=0,2\mu F$ . Find a) the electric current in the circuit b) charge of the capacitor?



picture 4

6. In the circuit (picture 5) we have  $\varepsilon_1=4V$ ,  $\varepsilon_2=72V$ ,  $r_1=r_2=1\Omega$ ,  $R_1=4k\Omega$ ,  $R_2=12k\Omega$ , and  $R_3=6\Omega$ . Find all the electric currents in the circuit?



7. In the circuit (picture 6) we have  $\varepsilon_1 = \varepsilon_2=2V$ ,  $r_1=1\Omega$ ,  $r_2=1,5\Omega$ , and  $R=1,4\Omega$ . Find:
- the potential difference at the ends of voltage sources  $\varepsilon_1$  and  $\varepsilon_2$
  - electric power at resistor  $R$ ?

